Regulating relative prices: mandated discounts and price menus*

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Abstract

Brazil requires cinemas to sell discounted tickets to students, the elderly, and other eligible groups at exactly half the posted full price. This regulation creates price discrimination with a government-chosen price ratio rather than a firm-chosen one. I study how such a constraint affects the design of price menus when firms can also offer voluntary promotional discounts to non-eligible consumers. In a monopoly model with three ticket categories, the mandate generate two mechanisms. First, it creates a cross-subsidy: the posted full price rises because the full-price ticket partly finances the regulated discount. Second, because the higher full price reduces the appeal of the regular ticket, the firm may deepen voluntary promotional discounts to redirect non-eligible consumers toward a lower-priced category. Under linear demand the mandate can generate higher total welfare than unconstrained third-degree price discrimination. Using a municipality-year panel of Brazilian cinemas from 2018 to 2023, I find evidence consistent with both mechanisms. Markets with a larger eligible share have higher posted full prices and deeper voluntary promotional discounts. I also show that about two-thirds of the cross-municipal variation in posted prices is explained by the national chain operating in each market rather than by local market conditions, suggesting that mandated discounts shape price menus partly through operator-level pricing rules.

Keywords: price discrimination, mandated discounts, cinema

JEL classification: L11, L82, Z11, D42, D21, D22

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1 Introduction

Price regulations often target a single transaction or consumer group, but firms usually choose prices as part of a broader menu. When a regulation fixes one relative price, firms may respond not only by changing the regulated price but also by redesigning unregulated prices and discounts. This paper studies such a response in the Brazilian cinema market, where federal law requires cinemas to sell tickets to students, elderly consumers, and other eligible groups at exactly half the posted full price (half-price law). The policy, therefore, creates price discrimination with a government-chosen ratio rather than a firm-chosen one. Because there is no government reimbursement, any adjustment to the posted full price is financed within the firm's own price menu, thus non-eligible consumers paying the full price may effectively subsidise eligible consumers receiving the mandated discount.[1]

The Brazilian half-price law provides an ideal setting for studying how firms organise prices under institutional constraints. In most markets, firms decide whether to price discriminate and how large a discount to offer. In Brazil, cinemas retain control over the posted full price and over voluntary promotions, but they do not choose the ratio between the full-price and half-price tickets. This creates a distinctive pricing problem: a firm that raises the posted full price also raises the regulated half-price ticket mechanically, while potentially making the regular full-price ticket less attractive to non-eligible consumers. The firm may therefore respond to the mandated ratio not only by increasing the posted price but also by expanding alternative discounted channels for non-eligible consumers.

Evidence that mandated discounts can inflate prices for non-eligible groups is available in other contexts. In Spain, residents of the Canary Islands, Balearic Islands, Ceuta, and Melilla receive a 75 percent discount on domestic airfares, with the government reimbursing airlines for the revenue shortfall.[2] Calzada and Fageda (2012) and Fageda et al. (2017) find that fares on subsidised routes are systematically higher than comparable unsubsidised ones. The Brazilian case differs because there is no

[1] Opinion articles in the public press argue this is the case. See https://exame.com/economia/por-que-a-meia-entrada-encarece-a-inteira/

[2] The stated objective for this subsidy is territorial cohesion.

government transfer. If a Brazilian cinema raises its full price in response to the mandate, it receives no reimbursement, and the cost falls within the consumer side of the market. This makes the policy a useful case for studying internal cross-subsidies and the organisational design of price menus.

I address these issues with a theoretical model of monopoly pricing under a mandated price ratio, disciplined by stylised facts from administrative data collected by the National Film Agency (ANCINE). The data reveal two features that motivate the model and the empirical analysis. First, cinemas use a three-tier pricing structure: the posted full price, the legally mandated half-price ticket, and voluntary promotional discounts, typically associated with credit-card partnerships or loyalty schemes. The promotional channel has expanded substantially over the period 2018–2025. Second, pricing is not purely local. Although cinemas operate in local markets, roughly two-thirds of the cross-municipal variation in posted full prices is explained by which national chain operates each market rather than by local market conditions. This suggests that regulated discounts are partly absorbed into operator-level pricing rules, a feature that connects the policy to the organisation of multi-market firms.

In the model, I consider three consumer groups: half-price eligible consumers, full-price consumers, and non-eligible consumers who can be reached through voluntary promotional pricing. I then characterise pricing and welfare under four regimes: (i) the mandate with the promotional channel, (ii) the mandate without promotion, (iii) uniform pricing, and (iv) voluntary third-degree price discrimination. The model identifies two main pricing mechanisms. The first is a cross-subsidy mechanism. This arises because the half-price ticket is mechanically tied to the posted full price, thus the firm raises the full price relative to the relevant unconstrained benchmark. The second is a cannibalisation mechanism. Once the full price is distorted upward, the firm has an incentive to deepen voluntary promotional discounts in order to redirect some non-eligible consumers away from the inflated full-price ticket and toward a lower-priced promotional tier. In this sense, the mandate reshapes the entire price menu, not only the regulated ticket category.

I then take the model's predictions to a municipality-year panel of Brazilian cinemas from 2018 to 2023. Consistent with the cross-subsidy mechanism predicted by the model, the data show that markets with a larger eligible share have higher posted full prices and higher average revenues per admission. They also have deeper voluntary promotional discounts, consistent with the cannibalisation mechanism predicted by the model. The chain-level pricing pattern further suggests that these responses are not only local adjustments to local demographics; national operators appear to use pricing rules that shape how the mandated discount is passed through across markets. To the best of my knowledge, this feature has not been previously discussed in the cinema-pricing literature (e.g., Davis 2006*a*; Einav 2007).

Related Literature. This paper contributes to three strands of the literature. First, it contributes to the theoretical literature on welfare under third-degree price discrimination (Schmalensee 1981; Varian 1985; Aguirre et al. 2010; Cowan 2012). This literature asks whether discrimination raises or lowers welfare relative to uniform pricing. Schmalensee (1981) shows that discrimination produces a net welfare loss unless it expands total output, while Aguirre et al. (2010) and Cowan (2012) establish that welfare and consumer-surplus rankings depend on demand curvature. I extend this question to the case of a mandated price ratio, where the firm does not freely choose the relative price and where an endogenous promotional tier can arise in response. The paper therefore shifts attention from voluntary price discrimination to regulated price discrimination with endogenous menu redesign.

Second, the paper relates to the literature on second-degree price discrimination and consumer self-selection (Mussa and Rosen 1978; Stole 2007). In that literature, firms design menus of price-access bundles and consumers sort according to their idiosyncratic types. The present model combines a third-degree regulatory constraint with a second-degree voluntary promotional channel. The firm is forced to offer a legally determined discount to eligible consumers, but it can still design voluntary discounts that induce non-eligible consumers to self-select across price tiers. This interaction between mandated and voluntary segmentation is central to the cannibalisation mechanism.

Third, the paper contributes to empirical work on cinema pricing and cultural-goods markets. de Roos and McKenzie (2014) study voluntary "Cheap Tuesday" discounts in Australian cinemas and find market expansion effects consistent with firms targeting price-sensitive consumers. I differ in studying a legally mandated ratio rather than a firm-chosen discount, and in documenting the endogenous promotional response that the mandate induces. On the supply side of cultural goods, Leslie (2004) estimates that price discrimination in a Broadway show raises firm's profits by roughly 5% relative to uniform pricing, with negligible consumer welfare effects; Courty and Pagliero (2012) finds a similar magnitude for concerts. Davis (2006*b*) and Einav (2007) pioneer discrete-choice analysis of film exhibition, with the former modelling geographic market power and the latter separating demand seasonality from film quality. A complementary supply-side literature studies cinema entry. Davis (2006*a*) measures business stealing, revenue cannibalisation, and market expansion when new theatres open in U.S. markets. My empirical contribution to this tradition is twofold. First, in the Brazilian setting, two-thirds of cross-municipal variation in posted prices is explained by chain identity rather than local market conditions, isolating a chain-level component of pricing variation that Davis (2006*b*) treats as a control. Second, the cannibalisation I document is demand-side across price tiers within a cinema, distinct from the supply-side across-location cannibalisation in Davis (2006*a*).

Within Brazil, Wink Junior et al. (2016) provides, to the best of my knowledge, the only prior empirical analysis of the half-price law, documenting demand-side effects on student cultural consumption. The present paper provides the first supply-side analysis, characterising how cinemas price in response to the mandate, how the promotional channel emerges endogenously, and how chain-level pricing structures the firm's response to local variation in eligibility.

The rest of the paper is organised as follows. Section 2 describes the institutional setting of the law. Section 3 presents stylised facts from the ANCINE data. Section 4 develops the theoretical model. Section 5 provides empirical evidence consistent with the model's main predictions, and Section 6 concludes. Proofs are in the appendix.

2 Half-price law in Brazil

Since the 1990s, at the regional level, Brazil has several laws establishing a minimum discount for cultural events. The first state law was established in the state of Bahia in 1990, and by 2008, when the state of Rio Grande do Sul established its law, all Brazilian states had adopted this type of legislation.

Besides state laws, different municipalities had their own local laws regarding this issue. For example, the city of Rio de Janeiro had such law since 1992, and the city of Porto Alegre, which is the capital of Rio Grande do Sul, created its own law in 2006, before the state established its own.

These regional laws varied in scope and targeted population. While all covered students, in some locations they were broader. For example, in the city of Rio de Janeiro teachers from public schools also had the right to the discount. However, it was not until December 2013 that a federal law was approved to discipline this issue.

The federal law was a reaction from pressure of several entities. According to the National Federation of Film Exhibition Companies (Feneec), in 2001 around 40% of tickets sold were at half-price, and this jumped to 70% by 2007. They argued one of the reasons for this was the expansion of fake student IDs.[3] Therefore, entities related to the film exhibition industry and other cultural activities started lobbying the government to regulate it, including establishing a quota for the amount of tickets that could be sold with a discount.[4]

This federal law guaranteed a 50% discount for cultural and sports activities for students, low-income people aged 15-29 years old, individuals older than 60 years old, and people with disabilities and their companion, if necessary.[5]

This law requires that, at least, 40% of all available tickets to be reserved for these groups. This constitutes a floor; exhibitors may voluntarily offer a larger share at the

[3] See https://www1.folha.uol.com.br/fsp/ilustrad/fq2904200707.htm

[4] See https://g1.globo.com/Noticias/Cinema/0,,MUL25098-7086,00.html

[5] Although the federal law does not contemplate them, in some of states blood donors also have the discount.

discounted rate. Nevertheless, in cinemas, this percentage is usually not binding and everyone who has the right to the discount buys the ticket without problem.[6]

Besides determining the value of the discount, and who has the right to it, the law also established the documents necessary to show that one has the right to the discount. Before the federal law, in the case of students, each educational establishment had to issue its own student ID. The new law determined that only IDs issued by the National Student Union (UNE), the National Student Union of Secondary School Students (UBES), and those by the National Association of Postgraduate Students (ANPG) are valid.[7] By centralising ID issuance, the reform aimed to reduce the number of people obtaining the discount without having the right to it.

Additionally, many cinemas launched their own fidelity card offering discounts, or they have agreements with other firms, and clients of these firms also have the discount. The most common case is that clients with a debit or credit card from a specific bank also have access to the discount. For example, clients from Bank Bradesco have 50% discount when buying a ticket at Cinemark. Other cinemas, like UCI, have their own relationship programme where consumers pay an annual fee and then have discounts in tickets or foods and beverages when attending a film session. As documented in Section 3, they have grown substantially over the period 2018–2025, and I incorporate them explicitly in the model presented in Section 4.

3 The cinema market in Brazil

I start by documenting stylised facts about the Brazilian cinema industry. The dataset comes from ANCINE for the period 2018-2025. It contains information from revenue and tickets sold at the municipality level, for each price category. Cinemas have to report this information for each session, and the aggregate data are available at ANCINE's webpage. However, the public available data do not contain the breakout by price category. I obtained this information through a FOI request, but due to legal

[6]This is not necessarily true for other cultural activities, such music festivals, which only sells the required 40% of tickets with discount.

[7]For non-students, other documents are valid, such as the national ID to show the 60+ condition, or documents issued by the authorities showing that a person is from low-income.

restrictions they could only provide the information at the city-year level.

With these data, I document six main facts. First, I present national aggregates in Table 1, and the share of each price category in Figure 1. The main fact is that only approximately 22% of tickets are sold at full price. Tickets sold at half-price represent approximately 58% of total tickets sold.

Additionally, Figure 1 reveals a steady rise in the share of tickets sold at promotional prices, increasing from 17% in 2018 to 24% in 2025. This increase is mirrored almost exactly by a decline in the full-price share over the same period, while the half-price share remains roughly stable. Two features of this trend are worth noting. First, the rise is gradual and persistent, suggesting it reflects a deliberate strategic response by exhibitors rather than a transitory shock. Second, the promotional channel is entirely voluntary, as exhibitors are under no legal obligation to offer it. This raises the question of why it has grown precisely in a market already subject to a mandated 50% discount.

Table 1. National aggregates, 2018–2025

Year	Number of Municipalities	Tickets (Thousands)	PMI (BRL)	Full-price (BRL)	Half-price (BRL)	% Full-price tickets	% Half-price tickets	% Promotional tickets
2018	405	164.4	14.89	21.82	12.80	26.5	54.2	17.6
2019	422	177.6	15.61	24.61	13.72	21.5	59.4	17.4
2020	426	39.9	15.88	25.44	14.00	20.0	61.0	17.3
2021	411	51.7	17.20	26.81	14.34	26.1	58.4	14.1
2022	423	97.9	18.80	29.38	16.39	23.4	59.6	15.4
2023	432	118.1	19.35	29.91	17.57	22.0	59.6	16.7
2024	447	129.2	19.58	30.96	18.14	19.9	57.4	20.8
2025	454	117.2	20.00	31.41	19.12	19.9	53.5	24.2

Note: PMI represents the average revenue per ticket. Full- and half price tickets are averages of all municipalities.

Second, I document the evolution of real ticket prices over the period. Figure 2 documents the evolution of ticket prices (full-price and voluntary promotion) over the period, and the average revenue per ticket (PMI). Panel A shows nominal prices while Panel B shows real values.[8] While nominal prices have risen moderately over the period, real ticket prices have declined throughout, suggesting that exhibitors have limited ability to pass through inflation to consumers. The promotional price tracks the full price closely, with the gap between the two remaining roughly stable

[8]Prices are deflated to 2019 BRL using the IPCA (the official inflation index). The series is obtained from the Central Bank's Time Series Management System - Series 13522.

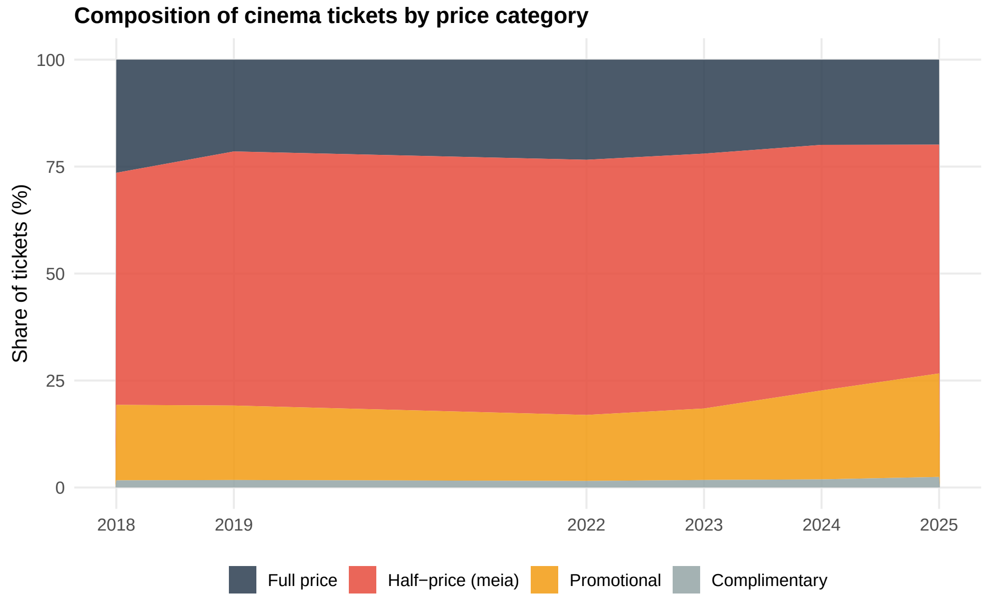

Figure 1. Share evolution: price categories
The Complimentary category represents tickets that were given to business partners, cinema critics, and/or tickets that were distributed for testing sessions.

in real terms. The growth in the promotional share documented in Figure 1 therefore reflects an expansion of the consumer base sorting into the promotional channel at a stable relative price, rather than a deepening of the discount itself.

Third, I document a composition effect. By law, the mandated price ratio between full and half-price ticket has to be 0.5. However, in Table 2 and Figure 3, I document that, for the year 2019, this is not the case.[9] This is expected because the dataset provides the total revenue, and the price I compute is a revenue-weighted average, which need not reproduce the mandated ratio at the individual-ticket level. For example, films with more tickets sold, such as Marvel films, will attract more audience with the right to half-price tickets, thus changing the composition in the data.

Table 2. Composition effect: half-price/full-price ratio (2019)

Mean ratio	Median ratio	SD ratio	P25	P75
0.58	0.509	0.203	0.497	0.569

Fourth, I document substantial cross-sectional variation in ticket prices and ticket composition across municipalities. Table 3 shows state-level summaries for 2019, illustrating that average full prices range from R$13.76 in the state of Acre to R$ 29.12 in

[9]The pattern is the same for other years.

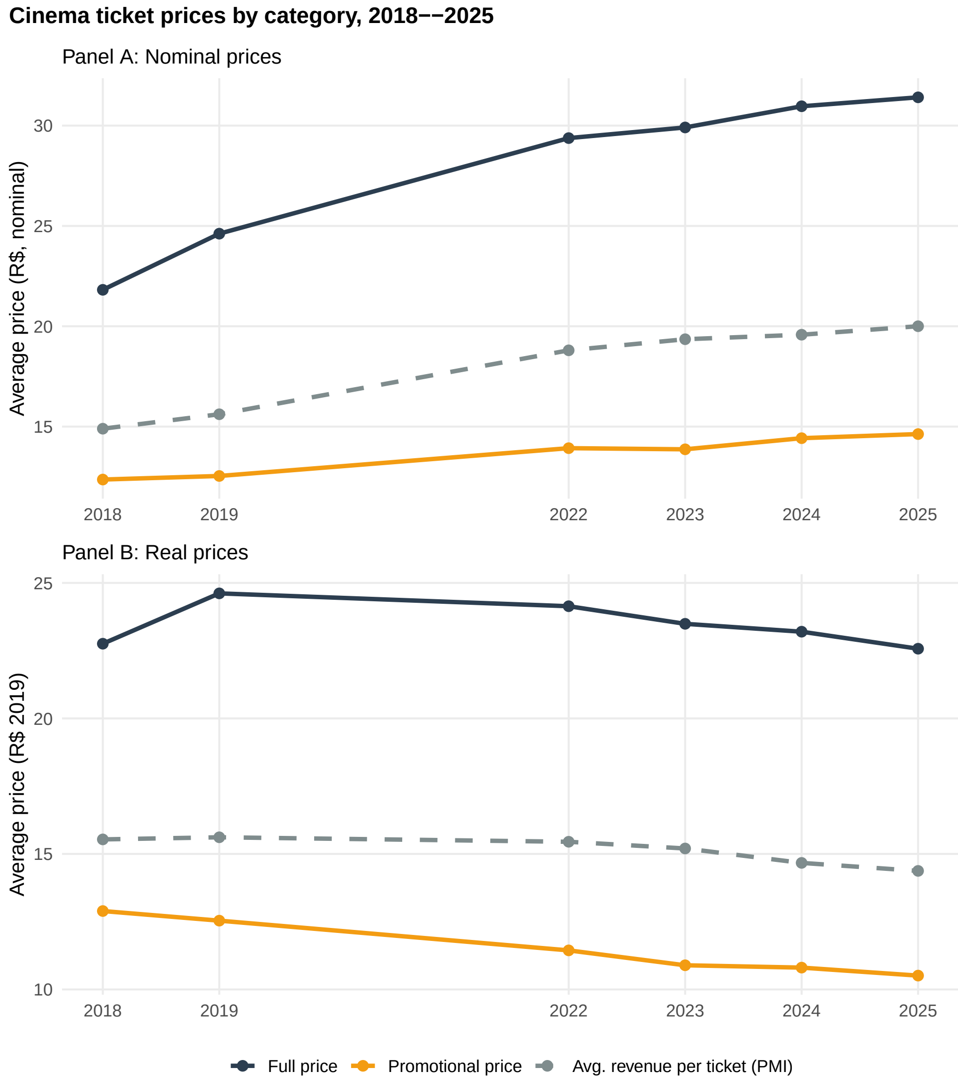

Figure 2. Cinema ticket prices by category, 2018–2025
Years 2020-2021 are excluded (COVID-19). Real prices (in terms of December 2019) deflated using IPCAs. Half price is omitted: by law it equals exactly half the full price at the point of sale.

the Federal District. Figure 4 plots the full price against the share of half-price tickets across municipalities. The relationship is weak and slightly negative, suggesting that markets with a higher share of half-price tickets sold do not systematically charge lower full prices.

The fifth fact is that most municipalities are served by monopoly or duopoly complex markets. Table 4 shows that around 70% of municipalities are monopolies, and this is stable over the whole period. Figure 5 shows the evolution of the number of screens and complexes over the period 2018-2025. With the exception of 2020[10], when the

[10]Year 2021 was still affected by the pandemic and it had restrictions on mobility.

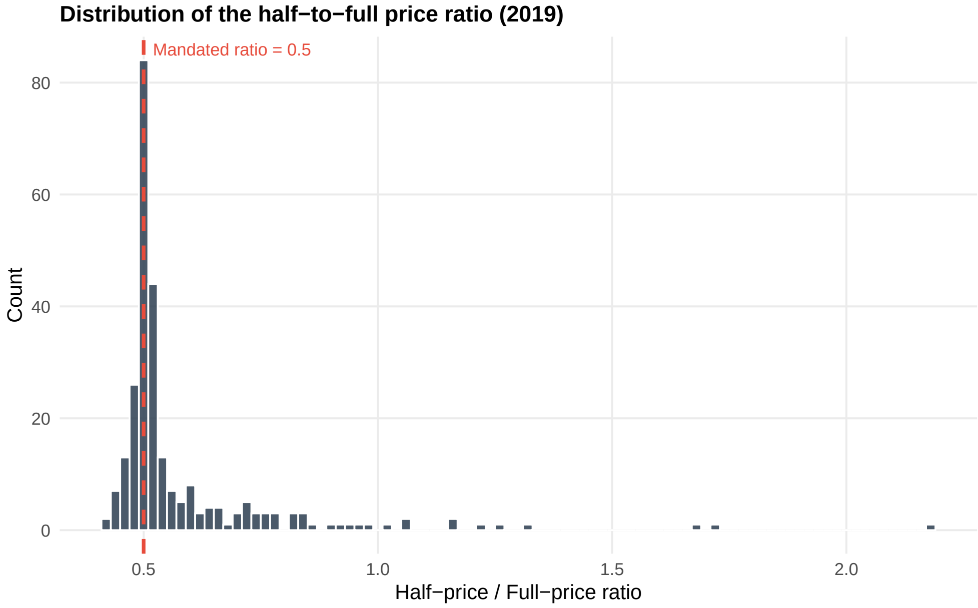

Figure 3. Ratio distribution

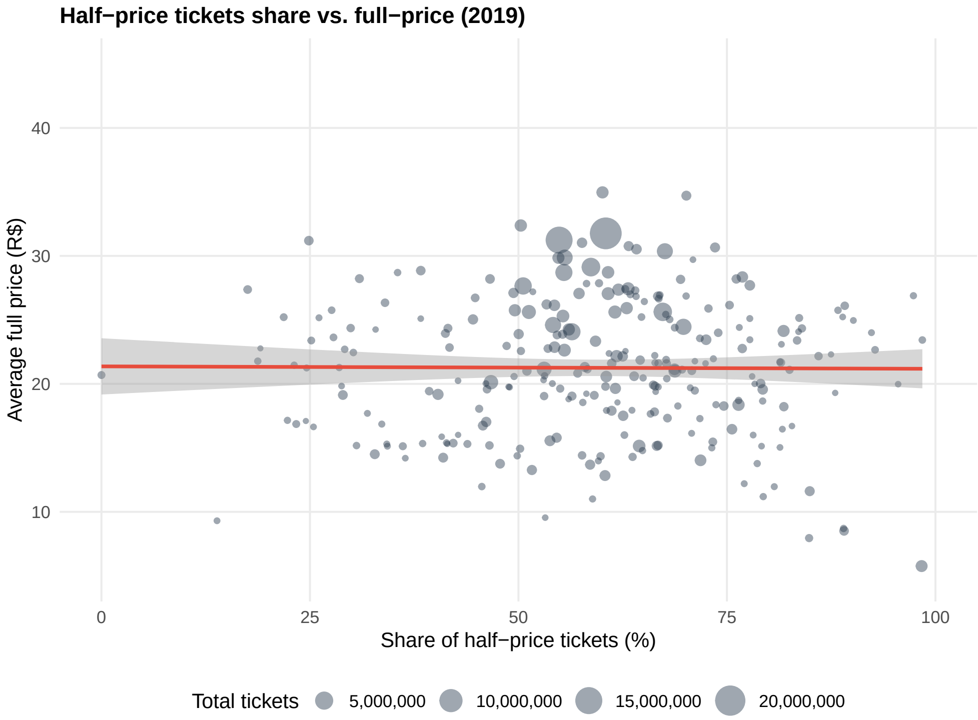

Figure 4. Relationship between average full-price and share of half-price tickets

industry was affected by the pandemic, the number of active screens and complexes is stable, thus entry and exit do not appear to be important features of this industry during the period under analysis.

A final fact concerns the pricing technology. Although each cinema typically operates as a local monopolist in its municipality (Fact 5), posted full prices vary across

Table 3. State-level summary (2019)

UF	N muni.	Tickets (M)	PMI (R$)	Inteira	Meia	% Meia	% Promo	Ratio
SP	71	58.83	16.74	26.32	14.58	62.5	13.6	0.554
RJ	16	23.23	16.70	26.99	15.38	56.7	25.7	0.570
MG	30	11.96	14.21	22.79	11.99	61.4	13.3	0.526
PR	19	9.30	14.62	23.64	12.72	57.2	19.9	0.538
RS	13	7.48	14.70	23.39	11.72	51.4	19.6	0.501
PE	12	6.21	15.95	27.31	14.35	58.6	21.2	0.525
BA	12	6.12	15.29	24.22	13.08	53.8	21.3	0.540
SC	21	5.89	15.99	25.91	12.68	56.2	13.4	0.489
DF	1	5.46	17.26	29.12	15.82	58.7	23.2	0.543
CE	5	4.92	15.44	23.73	13.17	55.8	16.2	0.555
GO	11	4.61	14.27	18.75	12.61	54.0	12.5	0.672
AM	1	3.55	14.25	24.46	12.46	69.8	9.4	0.510
PA	6	3.48	14.73	20.30	13.94	43.2	19.6	0.687
ES	8	3.27	12.81	20.34	11.52	64.4	16.1	0.567
MT	7	2.44	12.17	20.14	10.84	69.2	6.8	0.538
RN	2	1.93	15.28	24.67	13.50	59.1	17.9	0.547
MA	4	1.93	15.00	24.16	13.03	58.8	17.7	0.539
PB	4	1.72	15.39	28.14	14.32	53.5	30.7	0.509
MS	2	1.63	15.46	25.31	13.45	66.0	9.8	0.531
PI	3	1.47	12.49	22.65	11.22	76.3	7.9	0.495
AL	1	1.35	15.05	24.25	13.86	56.0	23.6	0.571
SE	3	1.23	14.29	25.87	13.76	69.2	18.5	0.532
RO	4	0.84	14.14	16.48	13.95	48.0	26.0	0.846
RR	1	0.74	12.41	16.46	11.93	75.6	9.9	0.725
AP	1	0.63	13.71	21.31	10.66	58.0	7.5	0.500
TO	2	0.62	12.52	16.87	10.59	50.3	9.6	0.628
AC	1	0.52	15.02	13.76	15.99	47.8	14.6	1.162

markets primarily because different national chains price differently, not because each market responds to local conditions. Regressing log of full price on chain-by-year fixed effects alone yields $R^2 = 0.66$ in the monopoly subsample, and $R^2 = 0.62$ in the full panel.[11] Therefore, roughly two-thirds of cross-municipal variation in

[11] In municipality-years with multiple chains operating, each municipality is assigned to a single dominant chain, defined as the chain operating the largest number of screens in that municipality-year (with ties broken alphabetically). The assignment is unambiguous in the majority of cases. In over half of municipality-years a single chain operates all screens, and the dominant-chain share of screens is at least 82% at the 25th percentile and 88% on average.

Table 4. Market structure by year

Year	Screens	Complexes	Munis.	Monopoly	% Monopoly
2018	3281	784	396	282	71.2
2019	3447	822	414	292	70.5
2020	1825	446	272	202	74.3
2021	3238	748	402	290	72.1
2022	3378	793	419	300	71.6
2023	3434	823	430	305	70.9
2024	3499	856	443	317	71.6
2025	3516	868	447	317	70.9

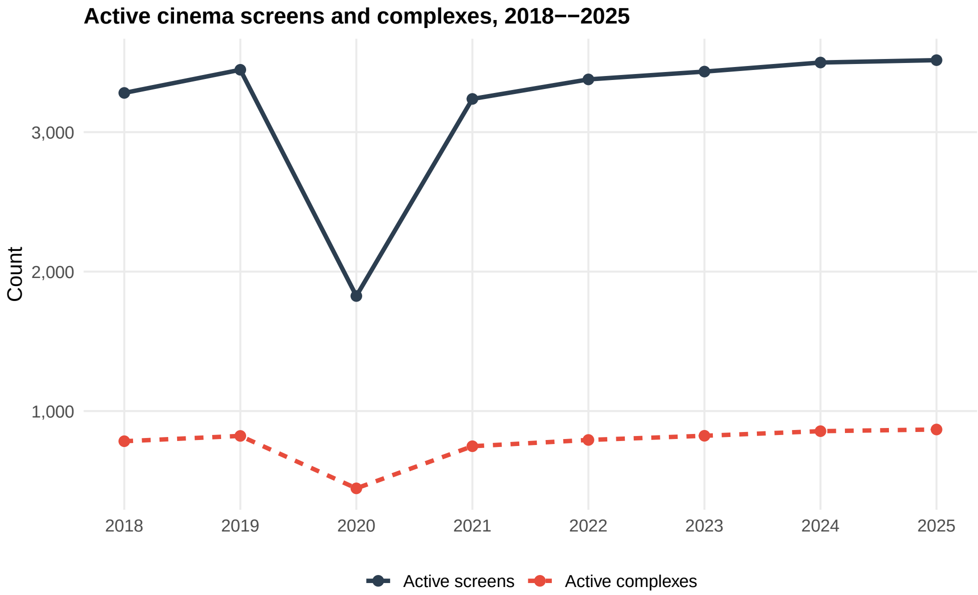

Figure 5. Screens and complexes evolution

posted full prices is explained by which national chain operates each market in a given year, indicating that prices are set partly at the operator level.

This decomposition of cross-market pricing variation into chain-level and market-level components has not been a focus of the prior empirical cinema-pricing literature (e.g., Davis 2006*b*; Einav 2007). The implication, developed further in the empirical analysis, is that prior models that treat local-market characteristics as the primary driver of cinema prices may be missing an important institutional feature.

4 Model

In this section, I present a model that captures the half-price law. I start with a simple setting where firms only offer the tickets with the required discount or with full price. Then, I extend it to incorporate the use of other promotional tickets in line with the findings in Section 3. Uniform pricing in cinemas is well documented in the literature (e.g. Orbach and Einav 2007), making the Brazilian half-price mandate an unusually explicit form of price discrimination.[12]

4.1 Notation and conventions

The full model considers three consumer groups: those who pay the full price (F), those eligible for the mandated half-price (H), and those who switch to the firm's voluntary promotional channel (P). Let N denote total market size, N_j the number of consumers in group j, and $\alpha_j = N_j/N$ the corresponding population share, with $\alpha_F + \alpha_H + \alpha_P = 1$ (and $\alpha_P = 0$ in the two-tier baseline).

In the linear demand setting, the per-capita demand of group j at price p_j is $q_j(p_j) = \bar{a}_j - \bar{b}_j p_j$, where $\bar{a}_j$ and $\bar{b}_j$ are the per-capita intercept and slope. Group-level (aggregate) demand follows from scaling by group size: $Q_j(p_j) = N_j q_j(p_j) = a_j - b_j p_j, a_j = N_j \bar{a}_j, b_j = N_j \bar{b}_j$. I use whichever form is more transparent in each derivation.

Marginal cost is normalised to zero to isolate the pricing effects of the mandated ratio and to yield closed-form welfare comparisons.[13]

4.2 Baseline two-tier model

Suppose each cinema is a local monopolist and has to choose its posted tickets' full price p. There is a group of consumers, N_F, that has no right to the discounted price, and faces this price p.

[12] Unlike other cultural markets, such as concerts and sports events, which have different prices for different seats.

[13] The half-price law constrains gross prices rather than margins, so $c > 0$ is not a harmless additive shift. The same economic forces remain, but exact formulas and welfare rankings depend on c.

By law, a group of consumers of mass N_H pays $p/2$. The firm cannot set different prices for each group independently, thus it controls only a single instrument, p, and the law mechanically determines the discounted price.

The total population is, therefore, $N = N_F + N_H$, and the half-price eligible share is $\alpha_H = N_H/N$, so that $N_F = (1 - \alpha_H)N$ and $N_H = \alpha_H N$.

Each group has linear demand as follows:

$$\begin{aligned} q_F(p) &= a_F - b_F \cdot p \\ q_H(p/2) &= a_H - b_H \cdot \frac{p}{2}, \end{aligned} \tag{1}$$

where $a_j > 0$ is the (aggregate) demand intercept for group j, capturing both market size and average willingness to pay, and $b_j > 0$ is the slope of the demand curve for group j. The half-price group is assumed to be more price-elastic in practice (e.g. students usually have lower income), which I parameterise in per-capita terms as $\bar{b}_H > \bar{b}_F$ and/or $\bar{a}_H < \bar{a}_F$ (equivalently, using the per-capita decomposition from Section 4.1, in aggregate terms, $b_H/N_H > b_F/N_F$).

Each cinema has the following profit function:

$$\pi(p) = p\,(a_F - b_F\,p) + \frac{p}{2}\left(a_H - \frac{b_H\,p}{2}\right). \tag{2}$$

The firm maximises its profit, which yields the following equilibrium price:

$$p^*_{HL} = \frac{2a_F + a_H}{4b_F + b_H}, \tag{3}$$

where the subscript HL stands for "half-price law." The firm chooses a single price, but the effective price vector is $(p^*_{HL},\ p^*_{HL}/2)$. An interior solution requires $p^*_{HL} <$

a_F/b_F and $p^*_{HL}/2 < a_H/b_H$, so that demand is positive for both groups at the optimum.

4.3 Benchmark Scenarios

4.3.1 Uniform Pricing

As a benchmark, I first analyse the case with uniform pricing (i.e. no price discrimination, either from the law or voluntary). In this case, the firm charges a single price p to everyone. Total demand is $Q(p) = (a_F + a_H) - (b_F + b_H)\,p$. The optimal price is:

$$p^*_U = \frac{a_F + a_H}{2(b_F + b_H)}. \tag{4}$$

Throughout the analysis, I assume that the optimal uniform price serves both groups, i.e. $p^*_U < \min(a_F/b_F,\ a_H/b_H)$, so that neither group's demand is driven to zero.

4.3.2 Voluntary Third-Degree Price Discrimination

In case there is no law, but the firm engages in third-degree price discrimination, it chooses p_F and p_H independently. Each group is, therefore, a standard monopoly problem:

$$p^*_F = \frac{a_F}{2b_F}, \qquad p^*_H = \frac{a_H}{2b_H}. \tag{5}$$

The voluntary discount ratio is:

$$r^* \equiv \frac{p^*_H}{p^*_F} = \frac{a_H b_F}{a_F b_H}. \tag{6}$$

Note that r^* is invariant to the aggregate vs per-capita choice of convention: writing

$a_j = N_j \bar{a}_j$ and $b_j = N_j \bar{b}_j$, the N_j factors cancel, so $r^* = \bar{a}_H \bar{b}_F / (\bar{a}_F \bar{b}_H)$ depends only on per-capita demand primitives.

In the Brazilian setting, the law mandates $r = 1/2$. This leads to the following result:

PROPOSITION 1 (Binding constraint). *The mandated 50% discount is binding (i.e., differs from the voluntary optimum) whenever $r^* \neq 1/2$, where $r^* = (a_H b_F)/(a_F b_H)$. Specifically:*

(a) *If $r^* > 1/2$ (the voluntary discount is less generous), the law forces a larger discount than the firm would choose.*

(b) *If $r^* < 1/2$ (the voluntary discount is more generous), the law constrains the firm from offering an even deeper discount.*

(c) *If $r^* = 1/2$, the law coincides with voluntary pricing.*

Case (a) arises whenever the eligible group's unconstrained monopoly price exceeds half of the full-price group's monopoly price, $r^* > 1/2$. In the empirically natural subcase $1/2 < r^* < 1$, the eligible group would voluntarily receive a discount, but a less generous one than the mandated 50% discount. If $r^* > 1$, the firm would voluntarily charge the eligible group more than the full-price group, so the mandated discount is even more restrictive. Case (b) arises when $r^* < 1/2$, so the firm would voluntarily offer an even deeper discount. Case (c) is the knife-edge case where the law is non-binding ($r^* = 1/2$).

4.4 Comparative statics in the two-tier model: pass-through to the full price

The first set of results concerns how the full price depends on the share of eligible consumers. Using the per-capita parameters: $a_j = N_j \bar{a}_j$ and $b_j = N_j \bar{b}_j$, I have:

$$p^*_{HL}(\alpha_H) = \frac{2(1-\alpha_H)\bar{a}_F + \alpha_H \bar{a}_H}{4(1-\alpha_H)\bar{b}_F + \alpha_H \bar{b}_H}. \tag{7}$$

PROPOSITION 2 (Pass-through). *With heterogeneous per-capita parameters, define $A(\alpha_H) =$*

$2(1-\alpha_H)\bar{a}_F + \alpha_H \bar{a}_H$ *and* $B(\alpha_H) = 4(1-\alpha_H)\bar{b}_F + \alpha_H \bar{b}_H$. *Then:*

$$\text{sign}\left(\frac{\partial p^*_{HL}}{\partial \alpha_H}\right) = \text{sign}\Big[(\bar{a}_H - 2\bar{a}_F)B(\alpha_H) - A(\alpha_H)(\bar{b}_H - 4\bar{b}_F)\Big].$$

The sign depends on which case of Proposition 1 applies. In Case (a), $\partial p^*_{HL}/\partial \alpha_H > 0$: when more consumers pay half-price, the effective revenue per ticket falls, and the firm compensates by raising the posted price, i.e. a pass-through from the mandate to full-price consumers, which is a form of cross-subsidisation. In Case (b), $\partial p^*_{HL}/\partial \alpha_H < 0$: the eligible group is sufficiently large and price-sensitive that the firm instead lowers the full price to maximise volume at $p/2$. I test this proposition empirically in Section 5.

The sign condition is globally positive when per-capita parameters are identical, as the following corollary shows.

COROLLARY 1 (Homogeneous case). *If* $\bar{a}_F = \bar{a}_H = \bar{a}$ *and* $\bar{b}_F = \bar{b}_H = \bar{b}$, *the pass-through is always strictly positive:*

$$p^*_{HL}(\alpha_H) = \frac{\bar{a}(2-\alpha_H)}{\bar{b}(4-3\alpha_H)}, \qquad \frac{\partial p^*_{HL}}{\partial \alpha_H} = \frac{2\bar{a}}{\bar{b}(4-3\alpha_H)^2} > 0.$$

In the data, the PMI reported by ANCINE is:

$$\text{PMI} = \frac{p^* \cdot q_F(p^*) + (p^*/2) \cdot q_H(p^*/2)}{q_F(p^*) + q_H(p^*/2)}. \tag{8}$$

As α_H rises, two opposing forces act on the PMI. The full price moves (in a direction depending on the case) but the share of half-price tickets also rises, pushing PMI down. The net effect is ambiguous and depends on parameters.

4.5 Three-Tier Pricing with Promotional Discounts

As shown in Section 3, cinemas usually offer their own promotional tickets as well, which represents approximately 15–25% of cinema tickets in Brazil. This is a form of voluntary price discrimination that operates alongside the mandated half price. In this subsection I extend the baseline model to incorporate this third pricing tier.

Now, there are three consumer groups. The first group (F) has population N_F and pays price p, the second group (H) has population N_H and pays price $p/2$ as mandated by law, and group (P) has population N_P and pays price $p(1-d)$, where $d \in [0, 1)$ is the promotional discount rate chosen by the firm.

Total population is the sum of the three groups, i.e. $N = N_F + N_H + N_P$. Define the following shares: $\alpha_H = N_H/N$ (half-price eligible), $\alpha_P = N_P/N$ (promotion eligible), and $\alpha_F = 1 - \alpha_H - \alpha_P$.

Now, the firm has two instruments, the posted price p and the promotional discount rate d. Still assuming linear demand, the demand for each group is:

$$q_F(p) = a_F - b_F \cdot p \tag{9}$$

$$q_H(p/2) = a_H - b_H \cdot \frac{p}{2} \tag{10}$$

$$q_P(p(1-d)) = a_P - b_P \cdot p(1-d). \tag{11}$$

With this new choice, the firm maximises profit over (p, d), with profit being:

$$\pi(p, d) = p\, q_F(p) + \frac{p}{2}\, q_H(p/2) + p(1-d)\, q_P\big(p(1-d)\big). \tag{12}$$

The first-order conditions are:

$$\frac{\partial \pi}{\partial p} = (a_F - 2b_F p) + \frac{1}{2}(a_H - b_H p) + (1-d)\big(a_P - 2b_P p(1-d)\big) = 0 \tag{13}$$

$$\frac{\partial \pi}{\partial d} = -p\big(a_P - 2b_P p(1-d)\big) = 0. \tag{14}$$

From equation (14), since $p > 0$, this problem yields the following result:

$$a_P - 2b_P p(1-d) = 0 \quad \Longrightarrow \quad p(1-d) = \frac{a_P}{2b_P}. \tag{15}$$

This is the standard monopoly price for the promotional group. Define $p_P^* \equiv a_P/(2b_P)$ as the optimal promo price. Then:

$$d^* = 1 - \frac{p_P^*}{p}. \tag{16}$$

REMARK 1 (Separation result). *The optimal promotional price equals the voluntary monopoly price for that group, regardless of the half-price law. The promo channel allows the firm to price discriminate freely for the promotional group, "undoing" the constraint imposed by the law for at least one segment. This clean separation result relies on the linearity of demand, which causes the FOC terms to be additively separable. Under nonlinear demand this need not hold exactly, though the logit extension in Section 4.9 shows that analogous promotional-discount mechanisms can arise under local curvature conditions.*

Substituting (15) into the FOC for p (equation (13)), the term involving the promo group vanishes:

$$(a_F - 2b_F p) + \frac{1}{2}(a_H - b_H p) = 0. \tag{17}$$

This is structurally identical to the baseline two-group FOC. Therefore:

PROPOSITION 3 (Full price under three-tier pricing). *The optimal full price in the three-tier model is:*

$$p^*_{3T} = \frac{2a_F + a_H}{4b_F + b_H}.$$

The formula has the same structural form as the baseline half-price-law price p^*_{HL} in equation (3). However, the numerical value need not equal the baseline p^*_{HL}. If the promotional group is split off from the full-price group, the aggregate full-price demand curve changes from $F + P$ (in the baseline two-tier reading) to F alone (in the three-tier model), so a_F and b_F take different numerical values across the two regimes. Thus, p^*_{3T} equals the baseline p^*_{HL} only when the full-price aggregate demand curve is held fixed across the two regimes. In per-capita form with $\alpha_F = 1 - \alpha_H - \alpha_P$, the formula becomes $p^*_{3T} = (2\alpha_F\bar{a}_F + \alpha_H\bar{a}_H)/(4\alpha_F\bar{b}_F + \alpha_H\bar{b}_H)$, which makes the dependence on α_P explicit.

The insight from this result is that the promo group does not affect p^* conditional on the full-price-group aggregates because the firm prices the promo group at its unconstrained optimum via d^*.

This proposition directly yields the value for the promotional discount rate:

COROLLARY 2 (Promotional discount rate). *Let $p^I_{3T} \equiv (2a_F + a_H)/(4b_F + b_H)$ denote the interior full-price formula from Proposition 3. If the interior condition $p^I_{3T} > p^*_P$ holds, the firm offers an interior promotional discount*

$$d^* = 1 - \frac{p^*_P}{p^I_{3T}},$$

and the equilibrium full price is $p^*_{3T} = p^I_{3T}$.

If $p^I_{3T} \leq p^*_P$, the promotional constraint $d \geq 0$ binds at $d^* = 0$; the firm offers no promotion and the equilibrium full price is determined by the no-promo first-order condition with *groups F, P, and H all paying p.*

Conditional on being interior, the discount d^* is larger when p_{3T}^I is high (i.e., when the half-price law inflates the full price) and when p_P^* is low (i.e., when the promo group has high price sensitivity).

Throughout the rest of the paper I focus on the interior case, which is the empirically relevant regime given the observed promo activity (Section 3).

4.5.1 Comparative Statics in the Three-Tier Model

PROPOSITION 4 (Promo discount and the eligible share). *When $\partial p_{3T}^*/\partial \alpha_H > 0$, which inherits the sign of $\partial p_{HL}^*/\partial \alpha_H$ from Proposition 2, we are under Case (a) of Proposition 1, and the optimal promo discount d^* is increasing in α_H:*

$$\frac{\partial d^*}{\partial \alpha_H} = \frac{p_P^*}{(p_{3T}^*)^2} \cdot \frac{\partial p_{3T}^*}{\partial \alpha_H} > 0.$$

The intuition behind this result is that as the half-price eligible share grows, the firm raises the full price and simultaneously deepens the promotional discount to keep the promo group's effective price at its unconstrained optimum.

PROPOSITION 5 (Promo discount in Case (b)). *Under Case (b) of Proposition 1, $\partial p_{3T}^*/\partial \alpha_H < 0$ by Proposition 2. Hence, the optimal promo discount is decreasing in the eligible share:*

$$\frac{\partial d^*}{\partial \alpha_H} = \frac{p_P^*}{(p_{3T}^*)^2} \cdot \frac{\partial p_{3T}^*}{\partial \alpha_H} < 0.$$

This result implies that a larger eligible share lowers the full price, and the firm shrinks the promotional discount because a lower full price requires less discounting to keep the promo group at its unconstrained optimum p_P^*.

The two cases of Proposition 1, therefore, generate opposite-signed predictions. In Case (a), the full price and the promotional discount move together with α_H, whilst in Case (b), they move together but in the opposite direction.

Empirically, this implies that municipalities with a larger half-price eligible population should have both higher full prices and larger promotional discount rates (or, equivalently, a higher share of promotional tickets), conditional on being in Case (a).

Furthermore, the observed PMI in the three-tier model is

$$\text{PMI} = \frac{p \cdot q_F + \frac{p}{2} \cdot q_H + p(1-d) \cdot q_P}{q_F + q_H + q_P}. \tag{18}$$

Holding α_P and per-capita primitives fixed, the promotional price $p(1-d^*) = p_P^*$ is invariant to α_H. Thus, the direct pricing effect of α_H operates through the full-price and half-price tiers (as in the baseline model), while the promotional tier contributes mainly through its level and composition effects.

4.6 Cannibalization and Endogenous Promotional Access

The separation result in Proposition 3 relies on the assumption that full-price and promotional consumers are distinct populations. This assumption is violated if promotional discounts are typically available to consumers who would otherwise pay the full price. For example, a consumer who holds the relevant credit card chooses between paying the full price and activating the promotional discount. This creates a cannibalization problem. Offering promotional discounts erodes full-price revenue.[14]

With this in mind, I extend the model as follows. There are two types of consumers: half-price eligible (share α_H) and non-eligible (share $1-\alpha_H$). Among non-eligible consumers, each individual i faces a hassle cost $h_i \geq 0$ of accessing the promotional

[14] I call this mechanism 'cannibalization' in the standard sense that the firm's voluntary promotional channel diverts non-eligible consumers who would otherwise pay the full price, reducing inframarginal full-price revenue. This is a within-cinema, demand-side mechanism, distinct from the across-location, supply-side cannibalization documented by Davis (2006*a*) in the context of U.S. cinema entry.

channel.[15][16]

Let h_i be drawn from a distribution $G(\cdot)$ with density $g(\cdot)$ on $[0, \bar{h}]$. A non-eligible consumer uses the promotional channel if the discount exceeds the hassle cost, i.e. $d \cdot p > h_i$ (equivalently, if $h_i < d \cdot p$). The fraction of non-eligible consumers who switch to promo is therefore:

$$\phi(d, p) = G(d \cdot p), \tag{19}$$

with $\phi(0, p) = 0$ and ϕ increasing in both d and p. The remaining fraction, $1-\phi(d, p)$, pays the full price.

Assume all non-eligible consumers share the same per-capita demand $q_N(\tilde{p}) = a_N - b_N \tilde{p}$, since they differ only in access costs. Eligible consumers also have per-capita demand $q_H(p/2) = a_H - b_H(p/2)$. The standalone monopoly price for the non-eligible group is $p_N^* \equiv a_N/(2b_N)$, which coincides with the promo-group standalone price p_P^* of the separable model (Section 4.5) under the natural identification $a_P = a_N$, $b_P = b_N$.

The firm's profit, with shares multiplying per-capita terms (per the convention in Section 4.1), is:

$$\pi(p, d) = (1-\alpha_H)\left[\big(1-\phi(d, p)\big)\, p\, q_N(p) + \phi(d, p)\, p(1-d)\, q_N\big(p(1-d)\big)\right] + \alpha_H \frac{p}{2} q_H(p/2), \tag{20}$$

so that the non-eligible mass $(1-\alpha_H)$ contributes profit at the full price p with

[15] Examples include signing up for a bank card, downloading a cinema app, remembering to bring the card, or signing up for a cinema loyalty programme.

[16] The cannibalization mechanism shares structural features with the second-degree price discrimination of Mussa and Rosen (1978). The promotional channel acts as a self-selection menu where the firm sets the discount, d, to attract consumers whose hassle cost is below $d\,p$. See Stole (2007) for a survey of the price discrimination literature that covers both the third-degree mandated and second-degree voluntary mechanisms.

probability $1-\phi(d,p)$ and at the promotional price $p(1-d)$ with probability $\phi(d,p)$, while the eligible mass α_H pays $p/2$.

The first-order condition for d is:

$$\frac{\partial \pi}{\partial d} = (1-\alpha_H)\Bigg[\underbrace{-p\cdot q_N\big(p(1-d)\big)\cdot\phi(d,p)}_{\text{margin loss on promo sales}} + \underbrace{p(1-d)\cdot q_N'\big(p(1-d)\big)\cdot(-p)\cdot\phi(d,p)}_{\text{demand response of existing promo buyers}} + \underbrace{\Big[p(1-d)\,q_N\big(p(1-d)\big) - p\,q_N(p)\Big]\cdot\phi_d(d,p)}_{\text{cannibalization: promo profit gained − full-price profit lost}}\Bigg] = 0, \tag{21}$$

where $\phi_d = \partial\phi/\partial d = p\cdot g(d\,p)$ is the density of marginal switchers.

The critical term is the last one. Each marginal consumer who switches from full-price to promo generates promo profit $p(1-d)\,q_N(p(1-d))$ but destroys full-price profit $p\,q_N(p)$. Since $p(1-d) < p$ and $q_N(p(1-d)) > q_N(p)$, the sign depends on whether the quantity gain outweighs the price-margin loss. The answer hinges on whether the half-price law distorts the full price above the standalone monopoly optimum for non-eligible consumers.

PROPOSITION 6 (Full price exceeds non-eligible standalone monopoly). *Let $p_N^* = a_N/(2b_N)$ denote the standalone monopoly price for the non-eligible group. Under the half-price law, $p_{HL}^* > p_N^*$ if and only if:*

$$\frac{a_H}{b_H} > \frac{a_N}{2b_N}.$$

This condition holds in Case (a) of Proposition 1. With identical per-capita parameters ($a_N = a_H$, $b_N = b_H$), it holds trivially.

The intuition behind this result is as follows. Because the half-price law forces the firm to serve the half-price group through the same posted price, the full price is distorted upward relative to what the firm would charge the non-eligible group in isolation. This means that the non-eligible segment generates lower revenue per potential buyer at the inflated full price than at its standalone monopoly price. Converting consumers to the promotional channel at a price closer to the standalone optimum is therefore revenue-enhancing.

In Case (b) of Proposition 1, this result does not hold, as summarised below.

COROLLARY 3 (Cannibalization mechanism in Case (b)). *In Case (b) of Proposition 1, the inequality of Proposition 6 reverses, i.e.* $p^*_{HL} < p^*_N$.

The firm's posted full price lies below the non-eligible standalone monopoly price, so converting non-eligible consumers to a promotional channel, which further reduces their effective price, would lower per-consumer revenue from the non-eligible segment rather than raise it. Thus, the cannibalization mechanism that motivates a deeper voluntary discount in Case (a) does not operate in Case (b).

Formally, the condition $a_H/b_H > a_N/(2b_N)$ says the eligible group's choke full-price equivalent is sufficiently high relative to the non-eligible monopoly price, i.e. the eligible group has sufficiently high demand that a deeper promo discount is revenue-enhancing.

On the promotional discount, cannibalization deepens it. The result is sharpest when stated holding the full price fixed, and the equilibrium comparison then follows from the price comparison below.

PROPOSITION 7 (Cannibalization deepens the promo discount). *Suppose* $p^*_{HL} > p^*_N$ *(Proposition 6).*

(i) ***Conditional on the full price.*** *For any full price* $p \in (p^*_N, a_N/b_N)$ *such that* $p - p^*_N \in (0, \bar{h})$ *and* $g(p - p^*_N) > 0$ *(so that the linear demand expression remains valid and small perturbations of d around the separable benchmark induce strictly positive marginal switching), allowing non-eligible consumers to self-select into the promotional channel induces a strictly deeper discount than the separable benchmark at the same* p: $d^*_C(p) > d^*_S(p) = 1 - p^*_N/p$, *with* $p(1 - d^*_C(p)) < p^*_N$.

(ii) ***Equilibrium comparison.*** *If, in addition, the separable three-tier benchmark is* *interior,* $p^*_{3T} > p^*_N$*, and the cannibalization equilibrium satisfies* $p^*_C \in (p^*_N, a_N/b_N)$ *(so it remains in the interior demand region) and* $p^*_C \geq p^*_{3T}$*, then* $d^*_C \geq d^*_{3T}$*, where* $d^*_{3T} = 1 - p^*_N/p^*_{3T}$ *is the separable-model equilibrium discount.*

At first glance, the conditional on p result may seem counterintuitive. Why does the firm want to cannibalise its own full-price sales? The answer lies in the half-price law's distortion. Because the half-price group pulls the full price above the standalone optimum for non-eligible consumers, the non-eligible segment generates lower profit contribution per potential buyer at the full price than at its profit maximising group price. Promotional access allows the firm to partially "undo" this distortion by routing non-eligible consumers to a price closer to their group optimum. The optimal promotional price can lie below the standalone monopoly price, $p(1 - d^*_C) < p^*_N$, because attracting additional marginal switchers, even those with relatively high hassle costs, can offset the lower margin earned on inframarginal promotional buyers.

Proposition 8 (Optimal discount in Case (b)). *Suppose Case (b) of Proposition 1 holds (so* $p^*_{HL} < p^*_N$ *by Corollary 3), with linear demand and G supported on* $[0, \bar{h}]$*. Then the firm's optimal promotional discount in the cannibalization model is a corner solution:* $d^*_C = 0$*. No voluntary promotional channel is offered, the cannibalization model coincides with the baseline two-tier model, and the equilibrium full price reverts to* p^*_{HL}*.*

The next result establishes a comparative static in promotional access that ties the two parts together.

Proposition 9 (Promo access and the full price). *Treat* ϕ *as an exogenous reduced-form parameter (rather than the equilibrium object* $G(d\,p)$*) indexing how widely the promotional channel is available to non-eligible consumers. Along the interior branch that starts from* $p^*_{HL} > p^*_N$ *(Proposition 6) and remains in the interior demand region, the re-optimised full price* $p^*_C(\phi)$ *is increasing in* ϕ*, and satisfies:*

$$\lim_{\phi \to 0} p^*_C(\phi) = p^*_{HL}, \qquad \lim_{\phi \to 1} p^*_C(\phi) = \frac{a_H}{b_H},$$

where the right-hand limit is the price that maximises eligible-group revenue $(p/2)(a_H - b_H p/2)$.

If the interior branch remains valid up to $\phi = 1$ (which requires $a_H/b_H < a_N/b_N$, so that the limiting price remains below the non-eligible choke price and the linear demand expression for the residual $1 - \phi$ mass of full-price non-eligible consumers stays valid), the limiting price is a_H/b_H. Otherwise, boundary or choke-price cases intervene before the limit is reached. In particular, along the interior branch where it is valid, $p_C^*(\phi) > p_{HL}^*$ for $\phi > 0$, i.e. an increase in promotional access raises the optimal full price.

REMARK 2 (Promo access in Case (b)). *In Case (b), the firm sets* $d_C^* = 0$ *(Proposition 8), so the promotional channel carries zero non-eligible mass and the comparative static in* ϕ *collapses to* $p_C^*(\phi) = p_{HL}^*$ *for all* ϕ*. The cannibalization model is degenerate in Case (b), reducing to the baseline two-tier model.*

The result should be read as a comparative static in promotional access, not as a fully-solved (p, d) equilibrium of the original $\phi(d, p) = G(d\,p)$ model. The fully endogenous model has no closed-form solution and has to be solved numerically.

This setup generates three sets of predictions relative to the separable benchmark. First, it implies deeper promotional discounts. The firm actively encourages non-eligible consumers to switch from full price to promo because the full price is distorted upward by the half-price law. Routing non-eligible consumers to the promotional channel raises expected profit contribution per potential buyer relative to leaving them at the inflated full price. The optimal promotional price can lie below the standalone monopoly price, $p(1 - d_C^*) < p_N^*$, because attracting additional marginal switchers can offset the lower margin on inframarginal promotional buyers.

Second, this yields an endogenous promotion expansion over time. As the half-price eligible share α_H grows, the full price rises further above the standalone optimum, increasing the revenue gain from each switcher. The firm responds by expanding promo access. This is consistent with the observed trend that the promo share rose from 17.6% in 2018 to 24.2% in 2025 while the full-price share fell from 26.5% to

19.9%.

Third, the full price rises with promo expansion. As more non-eligible consumers migrate to promo, the full price is increasingly determined by the half-price group. Since the half-price-optimal full price is higher than the baseline (which balances both groups), p_C^* rises with the access parameter. This creates a feedback loop in the fully endogenous model: higher full price $\to$ more incentive to switch to promo $\to$ higher full price. The equilibrium balances this feedback through the hassle cost distribution.

REMARK 3 (Nesting). *A restricted version of the separable model (Section 4.5) is nested as the limiting case in which a fixed fraction α_P of non-eligible consumers has zero access cost and the rest has prohibitively high access cost, with the promotional and full-price non-eligible subgroups sharing the same demand primitives ($a_P = a_N$, $b_P = b_N$). The unrestricted separable model, in which P has its own primitives (a_P, b_P) potentially distinct from (a_N, b_N), is not a literal special case of the cannibalization model.*

4.7 Welfare Analysis

In this subsection, I present the welfare effects of the half-price law with the three-tier structure presented above. The welfare analysis follows the standard third-degree price discrimination framework (Schmalensee 1981; Varian 1985), modified to incorporate the constraint that the firm chooses a single price subject to the mandated ratio.

With linear demand $q_j = a_j - b_j p_j$, the inverse demand is $P_j(q) = (a_j - q)/b_j$, and total welfare from group j at price p_j is:

$$W_j = \int_0^{q_j} P_j(s)\, ds = \frac{a_j q_j}{b_j} - \frac{q_j^2}{2b_j} = \frac{a_j^2 - b_j^2 p_j^2}{2b_j}. \tag{22}$$

The first term $a_j^2/(2b_j)$ is the maximum feasible welfare (at $p_j = 0$) and is constant across regimes. Therefore, comparing welfare across regimes reduces to comparing the welfare loss $L_j = b_j p_j^2/2$. Total welfare is:

$$W = \sum_j \frac{a_j^2}{2b_j} - \underbrace{\sum_j \frac{b_j\, p_j^2}{2}}_{\equiv L \text{ (total loss)}} . \tag{23}$$

Welfare is higher when the total loss L is lower.

4.7.1 Welfare Losses Under Each Regime

Uniform pricing. Both groups face $p_U = (a_F + a_H)/\big(2(b_F + b_H)\big)$:

$$L_U = \frac{b_F + b_H}{2} \cdot p_U^2 = \frac{(a_F + a_H)^2}{8(b_F + b_H)}. \tag{24}$$

Half-price law. Group F faces p_{HL}, group H faces $p_{HL}/2$, with $p_{HL} = (2a_F + a_H)/(4b_F + b_H)$:

$$L_{HL} = \frac{b_F}{2} p_{HL}^2 + \frac{b_H}{2} \left(\frac{p_{HL}}{2}\right)^2 = \frac{4b_F + b_H}{8} p_{HL}^2 = \frac{(2a_F + a_H)^2}{8(4b_F + b_H)}. \tag{25}$$

Voluntary discrimination. Each group faces its monopoly price $p_F^* = a_F/(2b_F)$, $p_H^* = a_H/(2b_H)$:

$$L_V = \frac{b_F}{2} \left(\frac{a_F}{2b_F}\right)^2 + \frac{b_H}{2} \left(\frac{a_H}{2b_H}\right)^2 = \frac{a_F^2}{8b_F} + \frac{a_H^2}{8b_H}. \tag{26}$$

4.7.2 Half-Price Law vs. Voluntary Discrimination

PROPOSITION 10 (Welfare comparison under linear demand: HL vs. voluntary). *Under zero marginal cost, linear demand, and interior solutions, $W_{HL} \geq W_V$ for all admissible parameter values, with equality if and only if $a_F b_H = 2a_H b_F$ (i.e., the voluntary discount* *ratio $r^* = 1/2$).*

Under the conditions of this proposition, the half-price law achieves weakly higher total welfare than the profit-maximising voluntary discrimination scheme, despite constraining the price ratio to 2:1.

The intuition is as follows. Under voluntary discrimination, the firm sets each group's price at the monopoly level, creating deadweight loss proportional to $b_j(p_j^*)^2$ in each market. Under the half-price law, the constraint links the two prices. The quadratic structure of welfare losses means that the gain from moving one price closer to zero (the half-price side) outweighs the loss from moving the other price further from zero (the full-price side), precisely because the deadweight loss is convex in price.

The firm unambiguously loses, i.e. $\pi_V \geq \pi_{HL}$, since adding a constraint cannot increase profits. But the consumer surplus gain more than compensates, yielding a net welfare improvement.

COROLLARY 4 (Distributional effects: HL vs. Voluntary). *Under the half-price law relative to voluntary discrimination:*

- *(i) Firm profits fall: $\pi_{HL} \leq \pi_V$.*
- *(ii) Total consumer surplus weakly rises: $CS_{HL} \geq CS_V + (\pi_V - \pi_{HL}) \geq CS_V$, with strict inequality whenever the mandated price ratio binds (i.e., $r^* \neq 1/2$).*
- *(iii) The direction of surplus change for each group depends on how the constrained prices compare to the voluntary prices. If $r^* > 1/2$, group F is worse off ($p_{HL} > p_F^*$) and group H is better off ($p_{HL}/2 < p_H^*$).*

4.7.3 Half-Price Law vs. Uniform Pricing

PROPOSITION 11 (Welfare condition: HL vs. Uniform). *$W_{HL} > W_U$ if and only if:*

$$b_F\, a_H(4a_F + 3a_H) > b_H\, a_F(3a_F + 2a_H). \tag{27}$$

Equivalently, defining $\rho \equiv a_H/a_F$ (relative market size) and $\sigma \equiv b_H/b_F$ (relative price *sensitivity):*

$$\sigma < \frac{\rho(4+3\rho)}{3+2\rho} \equiv \bar{\sigma}(\rho).$$

The welfare loss representation, $L_j = b_j p_j^2/2$, and the exact threshold $\bar{\sigma}(\rho)$ rely on the quadratic structure of welfare losses under linear demand. The qualitative trade-off carries over to alternative demand specifications, but the sign and threshold are functional-form dependent (a point emphasized by Aguirre et al. 2010, for the unconstrained third-degree problem).

COROLLARY 5 (Identical per-capita primitives). *If the two groups have identical per-capita demand primitives ($\bar{a}_F = \bar{a}_H$, $\bar{b}_F = \bar{b}_H$), then under the aggregate convention $\rho = \sigma = N_H/N_F = \alpha_H/(1-\alpha_H)$. Since $\bar{\sigma}(\rho) > \rho$ for all $\rho > 0$ (equivalently, $\rho(4{+}3\rho)/(3{+}2\rho) > \rho \iff 1+\rho > 0$), the welfare condition $\sigma < \bar{\sigma}(\rho)$ holds for any eligible share $\alpha_H \in (0,1)$. The half-price law therefore improves welfare over uniform pricing for any α_H under identical per-capita demand.*

COROLLARY 6 (Output comparison). *Total output is higher under the HL than under uniform pricing if and only if:*

$$2a_H b_F > a_F b_H \quad \iff \quad \sigma < 2\rho. \tag{28}$$

Since $\bar{\sigma}(\rho) < 2\rho$ for all $\rho > 0$, the welfare condition is strictly more demanding than the output condition: higher output is necessary but not sufficient for welfare improvement.

The condition under which the half-price law improves on uniform pricing connects to the broader question of when price discrimination raises consumer surplus relative to uniform pricing (Cowan 2012), with the added wrinkle here that the firm's discrimination ratio is fixed by mandate rather than chosen optimally.

4.7.4 Three-Tier vs. Two-Tier

Because switchers in the cannibalization model incur a private hassle cost h_i, the welfare object used in the separable analysis ($W = CS + \pi$, with CS given by linear integrated demand and no access term) is incomplete. For the cannibalization model only, I define the net welfare object that subtracts the integrated hassle cost paid by switchers:

$$W_C^{\text{net}} \equiv CS + \pi - (1 - \alpha_H) \int_0^{dp} h \, dG(h), \tag{29}$$

where the integral is the per non-eligible consumer expected hassle cost paid by those who switch (those with $h_i < dp$). I interpret h_i as the cost of gaining access to the promotional channel, incurred before the attendance decision, hence it is paid by all consumers who switch into the promotional channel, not only by those who ultimately purchase a ticket. I interpret the switching rule $h_i < dp$ as a reduced-form participation condition: consumers who switch are precisely those for whom the monetary saving and associated promotional benefits exceed the access cost. The access-cost term in W_C^{net} therefore measures the net resource or utility cost of this privately chosen switching margin. In terms of welfare accounting, the separable three-tier model of Section 4.5 corresponds to a case with no costly switching margin, so the access-cost integral is zero and W_C^{net} coincides with the standard $W = CS + \pi$. Throughout the rest of this paper, "welfare" in the cannibalization context refers to W_C^{net}; in the baseline, uniform, voluntary, and separable three-tier contexts it refers to the standard $W = CS + \pi$. This choice keeps the linear-welfare propositions (which involve no costly switching margin) unchanged.

PROPOSITION 12 (Promotional channel and welfare: local results). *(i)* Separable three-tier, fixed-p result. *In the separable model of Section 4.5, hold the posted full price p fixed at a level $p \in (p_P^*, a_P/b_P)$, so that group P's demand is positive at the full price and the promotional discount is interior. Adding the promotional channel for group P moves P's effective price from p to $p_P^* = a_P/(2b_P)$, which is strictly closer* *to P's welfare-maximising price under linear demand. F and H are unaffected at fixed p, and the firm's profit weakly rises because $d > 0$ is chosen voluntarily. Hence at fixed p, total welfare $W = CS + \pi$ strictly rises with the addition of the promotional channel.*

(ii) Cannibalization, fixed-p result. *In the cannibalization model of Section 4.6, hold the posted full price p fixed and consider introducing the promotional channel. Net welfare W_C^{net} as defined in (29) weakly increases relative to no promotion, by three observations: (a) under the reduced-form interpretation of the switching rule $h_i < d\,p$, switchers are precisely those for whom the monetary saving and associated promotional benefits exceed the access cost, so each switcher obtains a strictly positive net private gain; (b) the firm voluntarily sets $d > 0$ only when its own profit is weakly higher; and (c) the eligible group is unaffected at fixed p. Hence at fixed p, the consumer-side net surplus and the firm's profit both weakly rise, so net welfare weakly increases locally.*

REMARK 4 (Equilibrium-level comparison is ambiguous). *The fixed-p results in Proposition 12 do not extend to a global equilibrium ranking $W_{3T}(p_{3T}^*) \gtrless W_{HL}(p_{HL}^*)$ or $W_C^{net}(p_C^*) \gtrless W_{HL}(p_{HL}^*)$, because the promotional channel changes the firm's optimal posted full price. In the separable model, if the promo group P would otherwise have pulled the full price down (because P contributes heavily to the aggregate demand slope), separating P raises the full price faced by the remaining F and H groups, and the welfare losses from those groups can exceed the gains to P. A simple numerical example illustrates.*

With per-capita primitives $a_F = a_H = 0.1$, $b_F = b_H = 0.2$, $a_P = 0.2$, $b_P = 2$, the aggregated full-price slope falls from $b_F + b_P = 2.2$ to $b_F = 0.2$ when P is split off, the optimal full price rises from $p_{HL}^* \approx 0.078$ to $p_{3T}^* = 0.30$, and the welfare loss roughly doubles ($L_{3T} \approx 0.0138 > L_{HL} \approx 0.0068$). In the cannibalization model, the same equilibrium-comparison caveat applies a fortiori (Proposition 9: $p_C^* > p_{HL}^*$ on the interior region). A global welfare comparison therefore requires numerical evaluation or additional sufficient conditions; the model does not deliver it as a theorem.

The magnitude of the local welfare gain in both cases depends on the size of the affected populations and, in the cannibalization model, on the curvature of the hassle distribution $G(\cdot)$.

4.7.5 Summary of Welfare Rankings

PROPOSITION 13 (Welfare ordering). *Let π_V denote profit under fully unconstrained third-degree price discrimination over all relevant groups present in each regime: two prices (p_F, p_H) when only F and H are present, and three prices (p_F, p_H, p_P) in the three-tier setting. Under the maintained assumptions of zero marginal cost, linear demand, and interior solutions:*

$$\textit{Profit:} \quad \pi_V \geq \pi_{3T} \geq \pi_{HL}, \qquad \pi_V \geq \pi_U \tag{30}$$

$$\textit{Welfare:} \quad W_{HL} \geq W_V \tag{31}$$

$$\textit{Welfare:} \quad W_{3T} \gtrless W_{HL} \quad (\textit{see Remark 4}) \tag{32}$$

$$\textit{Welfare:} \quad W_{HL} \gtrless W_U \quad (\textit{condition } (\textbf{??})) \tag{33}$$

The profit ranking follows from nesting: the voluntary regime allows the firm to choose all relevant prices independently, and the three-tier regime adds the promotional instrument d on top of the half-price law. When all three groups F, H, P are present, π_{HL} denotes the regime in which F and P face the common posted full price, so the comparison $\pi_{3T} \geq \pi_{HL}$ is between two regimes with the same underlying populations. The profit comparison between π_{HL} and π_U is parameter-dependent because half-price-law pricing and uniform pricing are single-instrument problems with non-nested constraints, neither nested in the other.

The welfare comparison $W_{HL} \geq W_V$ in the three-group case is obtained by aggregation: under the half-price law without promotions, F and P both face the common full price, so they aggregate into a single non-eligible demand with $a_N = a_F + a_P$ and $b_N = b_F + b_P$, and the two-group result of Proposition 10 applies to N and H. Allowing voluntary discrimination separately between F and P can only increase the voluntary-discrimination welfare loss relative to the aggregated benchmark, so the ranking is preserved. Throughout, "3T" refers to the separable three-tier model of Section 4.5; the cannibalization-model comparison $W_C^{\text{net}} \gtrless W_{HL}$ is similarly ambiguous (Remark 4).

4.8 Capacity Constraint Extension

If the cinema has K seats per screening and the unconstrained optimum yields $Q^* > K$, the firm raises p until $Q(p) = K$:

$$(a_F - b_F p) + \left(a_H - \frac{b_H p}{2}\right) = K \quad \Longrightarrow \quad p_K = \frac{a_F + a_H - K}{b_F + b_H/2}.$$

The constrained price is decreasing in K. Interpreting a_j, b_j through the per-capita decomposition in Section 4.1, its comparative static with respect to α_H depends on the same demand-primitives tradeoff as in the unconstrained model, but also on the capacity level. However, average cinema occupancy rates in Brazil are around 10–15%, so the capacity constraint is unlikely to bind for most screenings. This extension is most relevant for blockbuster opening weekends or small-market cinemas, and is presented here for completeness.

This extension assumes interior demand for both groups. Capacity-binding parameter regions with one group at its choke price are beyond the scope of this paper.

4.9 Model extension: logit demand

In the previous subsections I derived the model assuming linear demand. I relax this assumption by assuming binary logit demand. This extension shows that analogous pass-through and promotional-discount mechanisms can arise under nonlinear demand, although the sign of the comparative statics depends on local curvature conditions. Welfare comparisons no longer admit a global ranking. The full derivation is presented below.

4.9.1 Setup

There are two types of consumers: half-price eligible (share α_H) and non-eligible (share $1 - \alpha_H$). Each consumer faces a binary choice between attending (inside good) and not attending (outside good) a cinema session. Eligibility determines which price each consumer faces, and they then decide independently whether to attend.

The non-eligible population is a single demand segment that may end up paying one of two prices: the full price p, or the promotional price $p(1-d)$ if they switch to the promotional channel. I denote its demand primitives by (δ_N, β_N) and use the labels F and P only to indicate which price the consumer ends up paying. The eligible population, with primitives (δ_H, β_H), always faces $p/2$.

Indirect utility for consumer i in group j at price p_j is:

$$u_{ij} = \delta_j - \beta_j \, p_j + \varepsilon_{ij}, \qquad j \in \{N, H\}, \tag{34}$$

where $\delta_j > 0$ is the mean utility from attending, $\beta_j > 0$ is the price coefficient, and ε_{ij} is i.i.d. Type I extreme value. Normalising the outside good utility to zero, the attendance share at price p_j for group j is:

$$s_j(p_j) = \frac{\exp(\delta_j - \beta_j \, p_j)}{1 + \exp(\delta_j - \beta_j \, p_j)}. \tag{35}$$

Note that each group faces its own binary logit, so $s_j \in (0, 1)$ is the probability that a group-j consumer attends at price p_j. This is distinct from a multinomial logit in which consumers choose simultaneously among F, H, and P.

For non-eligible consumers, $s_N(p)$ denotes the attendance share at the full price and $s_N(p(1-d))$ the attendance share at the promotional price. The price structure mirrors the linear model:

$$p_F = p, \qquad p_H = \frac{p}{2}, \qquad p_P = p(1-d), \tag{36}$$

where $p > 0$ is the posted full price and $d \in [0, 1)$ is the promotional discount rate chosen by the firm. A fraction $\alpha_H \in (0, 1)$ of the total population is eligible for the mandated half-price ticket. The remaining $(1 - \alpha_H)$ are non-eligible and face either the full price or the promotional price, depending on whether they switch.

4.9.2 Switching technology

A non-eligible consumer i uses the promotional channel if the monetary saving dp exceeds their personal net access cost η_i. For tractability, I model switching as depending on the posted monetary saving dp, rather than on the full expected surplus difference $CS_N(p(1-d)) - CS_N(p)$. This captures reduced-form promotional salience and access incentives, and is consistent with the linear cannibalization setup of Section 4.6.

I model η_i as drawn from a logistic distribution with location $\mu > 0$ and scale $1/k$. The net cost η_i aggregates the hassle of accessing the channel (registering a bank card, downloading an app, joining a loyalty programme) with any non-monetary benefits (loyalty rewards, points, status). It is therefore not constrained to be non-negative. The mean friction is captured by $\mu > 0$, so on average switching has a positive cost, but a tail of consumers actively prefer the promotional channel for non-monetary reasons. This reduced-form smooth participation form is consistent with the cannibalization setup in Section 4.6, except that the hassle distribution is now allowed to place mass on negative values.

The share of non-eligible consumers who switch is:

$$\phi(d, p) = \Pr(\eta_i < dp) = \frac{1}{1 + \exp\big(-k(dp - \mu)\big)}, \tag{37}$$

where dp is the monetary saving from switching. The partial derivatives are:

$$\frac{\partial \phi}{\partial d} = k\,\phi(1-\phi)\; p > 0, \qquad \frac{\partial \phi}{\partial p} = k\,\phi(1-\phi)\, d > 0. \tag{38}$$

4.9.3 Demand allocation and firm problem

Quantities demanded by each group are:

$$q_H = \alpha_H \, s_H\left(\frac{p}{2}\right), \tag{39}$$

$$q_F = (1-\alpha_H)\,\big(1-\phi(d,\,p)\big)\,s_N(p), \tag{40}$$

$$q_P = (1-\alpha_H)\,\phi(d,\,p)\,s_N\big(p(1-d)\big), \tag{41}$$

where s_N is the common non-eligible attendance share function. The firm chooses (p, d) to maximise:

$$\pi(p, d) = \frac{p}{2}\,q_H + p\,q_F + p(1-d)\,q_P. \tag{42}$$

It is useful to define the logit marginal profit for group j at price p_j as:

$$M_j(p_j) \equiv \frac{d}{dp_j}\left[p_j\, s_j(p_j)\right] = s_j(p_j)\left[1-\beta_j\; p_j\;(1-s_j(p_j))\right]. \tag{43}$$

By standard properties of the logit, $M_j(p_j^*) = 0$ defines the unconstrained monopoly price for group j; $M_j(p_j) > 0$ if and only if $p_j < p_j^*$; and $M_j(p_j) < 0$ if and only if $p_j > p_j^*$. For the non-eligible group, $M_N(p)$ and $M_N(p(1-d))$ are evaluated at the same demand primitives.

4.9.4 Price pass-through under logit

PROPOSITION 14 (Pass-through under logit). *At an interior optimum (p^*, d^*) satisfying the second-order condition (Hessian negative definite), let $\Psi(p, d, \alpha_H) \equiv \partial\pi/\partial p = 0$ denote the first-order condition for p. Then:*

$$\operatorname{sign}\left(\frac{\partial p^*}{\partial \alpha_H}\right) = \operatorname{sign}\left(\tfrac{1}{2}\, M_H\left(\frac{p^*}{2}\right) - \widetilde{\Psi}_{F+P}(p^*, d^*)\right),$$

where $\widetilde{\Psi}_{F+P}(p, d) \equiv \dfrac{\partial}{\partial p}\Big[(1-\phi)\; p\, s_N(p) + \phi\; p(1-d)\, s_N\big(p(1-d)\big)\Big]$.

In particular, $\partial p^*/\partial \alpha_H > 0$ whenever $M_H(p^*/2) > 0$ and $\widetilde{\Psi}_{F+P}(p^*, d^*) \leq 0$, i.e., when the mandated half-price lies below the half-price group's unconstrained monopoly price ($p^*/2 < p^*_H$) and the non-eligible margin from the two price tiers, taken together, would benefit from a marginal price reduction.

Remark 5 (Relation to the linear case). *The condition $p^*/2 < p^*_H$ corresponds to the case $r^* > 1/2$ in the linear model (Proposition 1, Case (a)): the mandated discount is more generous than the voluntary optimum, so the law binds downward for group H.*

Remark 6 (Pass-through in logit Case (b)). *If $p^*/2 > p^*_H$ (the logit analog of Case (b), in which the mandated half-price exceeds the eligible group's unconstrained monopoly price), then $M_H(p^*/2) < 0$. The FOC (A.9) forces $\widetilde{\Psi}_{F+P}(p^*, d^*) > 0$, and:*

$$\operatorname{sign}\left(\frac{\partial p^*}{\partial \alpha_H}\right) = \operatorname{sign}\left(\tfrac{1}{2}\, M_H\left(\tfrac{p^*}{2}\right) - \widetilde{\Psi}_{F+P}(p^*, d^*)\right) < 0.$$

A larger eligible share lowers the equilibrium full price under logit, paralleling the Case (b) result in the linear model (Proposition 2).

4.9.5 Promotional discount under logit

Proposition 15 (Promotional discount under logit). *Suppose $\partial p^*/\partial \alpha_H > 0$, the second-order condition for d holds ($H_d \equiv \partial^2\pi/\partial d^2 < 0$), and the cross-partial $H_p \equiv \partial^2\pi/\partial d\,\partial p$ is positive at the equilibrium. Then the optimal promotional discount $d^*(\alpha_H)$ is strictly increasing in α_H.*

The positive cross-partial $H_p > 0$ is a maintained assumption rather than a primitive. Intuitively, when the full price p rises (because α_H has risen), the per-consumer profit gap between the promo and full-price tier widens whenever the full price exceeds the unconstrained non-eligible optimum, so the firm has a stronger marginal incentive to deepen the discount. The appendix formalises this as a local sufficient condition: if $H_p > 0$, $H_d < 0$, and $dp^*/d\alpha_H > 0$, then $dd^*/d\alpha_H > 0$.

PROPOSITION 16 (Promotional discount under logit, Case (b)). *Suppose* $\partial p^*/\partial\alpha_H < 0$ *(logit Case (b), Remark 6), the second-order condition for d holds* ($H_d < 0$)*, and the cross-partial* $H_p > 0$ *at the equilibrium. Then the optimal promotional discount* $d^*(\alpha_H)$ *is strictly decreasing in* α_H*.*

The economic content mirrors Case (a) with reversed signs. When the full price falls (because α_H has risen), the per-consumer profit gap between the promo and full-price tiers narrows, and the firm responds by shrinking the discount. The same local sufficient condition applies: under $H_p > 0$, $H_d < 0$, and $dp^*/d\alpha_H < 0$, the chain-rule expression delivers $dd^*/d\alpha_H < 0$.

4.9.6 Cannibalization under logit

PROPOSITION 17 (Cannibalization under logit). *Suppose the conditions of Proposition 15 hold* ($\partial p^*/\partial\alpha_H > 0$, $H_p > 0$, $H_d < 0$)*. Then the equilibrium switching share* $\phi(d^*(\alpha_H), p^*(\alpha_H))$ *is strictly increasing in* α_H*.*

PROPOSITION 18 (Cannibalization under logit, Case (b)). *Suppose the conditions of Proposition 16 hold* ($\partial p^*/\partial\alpha_H < 0$, $H_p > 0$, $H_d < 0$)*. Then the equilibrium switching share* $\phi(d^*(\alpha_H), p^*(\alpha_H))$ *is strictly decreasing in* α_H*.*

Mirroring the linear cannibalization model of Section 4.6, the equilibrium switching share moves in the same direction as the equilibrium full price: rising with α_H under Case (a) and falling under Case (b). The observed time trend in the data (see Section 3) is consistent with Case (a).

4.9.7 Welfare under logit

Under logit demand, the consumer surplus for group j at price p_j is the standard logit inclusive value:

$$CS_j(p_j) = \frac{1}{\beta_j} \ln\left(1 + \exp(\delta_j - \beta_j p_j)\right). \tag{44}$$

Switchers also incur the net access cost η_i. The expected access cost paid per non-eligible consumer who switches is $E[\eta \mid \eta < dp]$, so the total expected access cost across all switchers is $(1 - \alpha_H) \int_{-\infty}^{dp} \eta \, dG(\eta)$, where G is the logistic CDF in (37). Using N subscripts to denote the common non-eligible primitives, total consumer surplus and welfare are:

$$CS = \alpha_H \, CS_H\left(\frac{p}{2}\right) + (1 - \alpha_H)(1 - \phi) \, CS_N(p) + (1 - \alpha_H)\phi \, CS_N\big(p(1 - d)\big) \tag{45}$$
$$- (1 - \alpha_H) \int_{-\infty}^{dp} \eta \, dG(\eta),$$
$$W = CS + \pi. \tag{46}$$

The access-cost term adjusts consumer surplus for the non-price costs or benefits of using the promotional channel: switchers with $\eta_i > 0$ pay a hassle cost that reduces surplus, while switchers with $\eta_i < 0$ derive a non-price benefit (e.g. loyalty rewards) that adds to it. The sign of the integral therefore depends on the location of the logistic relative to zero and on the size of dp.

No global analytical welfare ranking is available across pricing regimes under logit demand (in line with Aguirre et al. 2010, on the role of demand curvature in welfare effects under unconstrained third-degree discrimination).

REMARK 7 (Welfare under logit). *Two qualitative directions follow from the structure of the model. If switching frictions are low and the promotional channel expands non-eligible output without excessive cannibalization of high-value full-price consumers, the half-price regime with promotions can dominate uniform pricing. Conversely, when switching is extensive and discounts are large, the induced changes in effective prices and access-cost payments can make the constrained-plus-promo regime less efficient than voluntary discrimination. Numerical comparison is required to determine which case obtains for any* *given parameterisation; the model does not deliver a global ranking.*

5 Empirical Evidence

This section tests the comparative predictions of Propositions 2 and 4. These propositions imply that monopoly markets with a larger eligible share should display a higher posted full price and a deeper voluntary promotional discount. Due to data availability, I estimate the relationship on a municipality-year panel of Brazilian cinema markets from 2018 until 2023. I find support for both propositions documenting positive correlations between the eligible share in a municipality and the full price, the average revenue per admission, and the promotional discount.

5.1 Data and sample

The ANCINE data described in Section 3 provide revenue and tickets by price category at the municipality-year level, from which I construct the average full price (p_F), the average revenue per admission (PMI), the voluntary promotional discount ($d = 1 - p_P/p_F$),[17] and the promotional share, defined as the fraction of paid tickets sold at the promotional price.

The sample is restricted to monopoly markets (which account for 71% of the panel, as described in Section 3) to match the model, and it excludes 2020 and 2021 because of pandemic disruption.

The eligible share, $\alpha_{i,t}$, is the share of municipality i's population in year t that satisfies at least one of the following law's eligibility criteria: age 60 or above, student of any level, age 15–29 with low income, or disability.

In my primary specification, eligible share is defined as following:

$$\alpha_{i,t} = \frac{\text{higher-education enrolment}_{i,t} + \text{population aged 60+}_{i,t}}{\text{total population}_{i,t}}.$$

[17] The discount d is the empirical analogue of the model's voluntary discount: $p_P = p_F(1-d)$ is the promotional price, where p_P computed as promotional revenue divided by promotional tickets sold in each municipality-year.

To provide robustness, I create two other measures. The first one, which I call α^{broad}, replaces higher-education enrolment with total enrolment (primary and secondary plus higher education), i.e. $\alpha^{\text{broad}}_{i,t} = \frac{\text{total enrolment}_{i,t} + \text{population aged 60+}_{i,t}}{\text{total population}_{i,t}}$.

The second measure, which I call α^{demog}, replaces enrolment data with the share of individuals aged 15-29, i.e. $\alpha^{\text{demog}}_{i,t} = \frac{\text{population aged 15–29}_{i,t} + \text{population aged 60+}_{i,t}}{\text{total population}_{i,t}}$. The law restricts youth eligibility to low-income individuals in this age range, but since I do not observe income at each age range, I use all individuals 15–29 and therefore captures more than the law's strict criterion.[18]

The age components are constructed from the 2022 population census,[19] interpolated to other panel years using PNAD-Contínua. PNAD is household survey representative at the state level, so I scale each municipality's 2022 age-bracket counts by the state-year age-bracket trend implied by PNAD.

The law also determines that individuals with disability and their companion have the right to half-price tickets, but I do not have municipality-level disability data. Nevertheless, according to the 2022 Census, approximately 45% of people with disabilities are aged 60 or above, so the bulk of this group is already captured in the elderly age component.

For the student information, I obtained education data from INEP Education Censuses. These datasets contain the number of students enrolled, every year, in each municipality at the basic (primary and secondary) and higher education level.

I use ANCINES' complex registry to identify the cinema chain that operates in each municipality. If no chain operates in a municipality, I classify it as an independent cinema. Additionally, from this source, I obtain the number of screens at each municipality.

Finally, from the Brazilian Institute of Economics and Statistics (IBGE) I obtain information about total population per municipality and GDP per capita per municipality.[20]

[18] Nevertheless, this age group contains many students, so it also captures that demographic if they are not low income.

[19] The 2022 Census reports counts in five-year brackets, from 0–4 up to 100+.

[20] Municipal GDP is available only until 2023, thus determining the final year of my analysis. I

5.2 Empirical strategy

Propositions 2 and 4 are statements about how outcomes vary with eligible share across markets, holding firm primitives fixed. I estimate

$$y_{i,t} = \beta\, \alpha_{i,t} + \mathbf{X}'_{i,t}\gamma + \mu_{c(i,t),t} + \varepsilon_{i,t}, \tag{47}$$

where i indexes municipalities, t indexes years, $c(i, t)$ is the chain in market i at year t, and $\mu_{c(i,t),t}$ is a chain-by-year fixed effect. The outcome y is log p_F, log PMI, the discount d, or the promo share. $\mathbf{X}$ is a vector of control variables. Standard errors are clustered by municipality.

Identification of β comes from variation in α across municipalities conditional on $\mu_{c(i,t),t}$, motivated by the chain-pricing pattern documented in Section 3. For comparison, I also report a specification with state-by-year fixed effects, and a two-way fixed-effects specification with municipality and year fixed effects.

5.3 Pass-through of the full price (Proposition 2)

Tables 5 and 6 report estimates of equation (47) for log p_F and log PMI, respectively. The main specification of interest is in column (4), and it shows a positive coefficient for both outcomes. These estimates imply that a one percentage point higher eligible share is associated with a 0.93% higher full price and a 0.44% higher average revenue per admission, conditional on chain-by-year fixed effects. The signs match Case (a) of Proposition 1. This suggests that revenue from each additional eligible buyer at the mandated half price falls short of the full-price revenue it displaces, and the firm partially recovers the difference through the posted price, providing evidence supporting the existence of the cross-subsidy mechanism.

Columns (1) and (2) report, respectively, specifications with state-by-year (between) and municipality and year fixed effects (within). As expected, both specifications

estimate the same regressions described in Section 5.2 without this variable, and thus adding the year 2024 to the sample. Results are qualitatively the same and are available upon request.

Table 5. Proposition 2: eligible share and full price

	Dep. variable: log full price		
	(1) between	(3) TWFE	(4) chain-year
Eligible share	0.522	0.961	0.925***
	(0.559)	(1.598)	(0.278)
log income p.c.	−0.013	0.114	0.037
	(0.038)	(0.163)	(0.028)
log population	−0.034	−0.496	−0.042*
	(0.032)	(0.308)	(0.024)
Screens	0.053***	0.031	0.035***
	(0.009)	(0.032)	(0.006)
N	1036	1036	1036
Within R^2	0.269	0.052	0.216

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. Column (3) contains chain-year fixed effects.

Table 6. Proposition 2 (PMI): eligible share and average revenue per admission

	Dep. variable: PMI		
	(1) between	(3) TWFE	(4) chain-year
Eligible share	0.319	0.042	0.436**
	(0.367)	(0.851)	(0.210)
log income p.c.	0.036	0.117*	0.042**
	(0.027)	(0.070)	(0.020)
log population	−0.001	−0.108	−0.059**
	(0.022)	(0.154)	(0.025)
Screens	0.041***	−0.002	0.034***
	(0.007)	(0.016)	(0.006)
N	1036	1036	1036
Within R^2	0.348	0.021	0.223

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. Column (3) contains chain-year fixed effects.

yield estimates that are not statistically distinguishable from zero. The between estimation pools the marginal response to the eligible share with cross-chain price differences, but in Section 3 I showed that chain-by-year fixed effects absorb two-thirds of the variation in log p_F across municipalities, thus there is not enough variation to be explained by the between specification. The two-way fixed effects

Table 7. Heterogeneous pass-through by GDP tercile

	log full price	PMI
Eligible share × low income	0.853***	0.338
	(0.294)	(0.214)
Eligible share × mid income	0.911***	0.474**
	(0.265)	(0.213)
Eligible share × high income	1.025***	0.431*
	(0.313)	(0.227)
N	1036	1036
Within R^2	0.218	0.227

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. The specification conditions on chain-by-year fixed effects.

specification relies on within-municipality variation in α, which is small for a variable driven primarily by the slowly evolving age structure.

5.4 Heterogeneity

In this subsection, I split the sample based on the municipality income (measure by GDP per capita) and estimate the pass-through separately by income tercile. I present the results in Table 7. The point estimates are roughly constant across terciles for the full price, and positive across terciles for the PMI with the low-income PMI estimate not being distinguishable from zero at a conventional significance level. The qualitative pattern is consistent with Case (a) of Proposition 1 holding throughout the sample rather than only in a subset of markets.

5.5 Promotional response (Proposition 4)

Tables 8 and 9 report estimates with the voluntary promotional discount d and the promotion share as outcomes, respectively. Notice that they differ in sample size. The discount regression includes only the market-years in which the firm sold at least one promotional ticket.

The results suggest that a one percentage point higher eligible share is associated with a 0.30 p.p. deeper voluntary discount. The promotion share is not significantly

Table 8. Proposition 4: eligible share and the voluntary promotional discount

	Dep. variable: Promo discount		
	(1) between	(2) TWFE	(3) chain-year
Eligible share	0.098	−0.034	0.297**
	(0.186)	(1.179)	(0.117)
log income p.c.	−0.020	0.098	0.004
	(0.021)	(0.085)	(0.012)
log population	−0.016	−0.199	−0.004
	(0.019)	(0.171)	(0.015)
Screens	0.010**	0.017	0.002
	(0.004)	(0.013)	(0.003)
N	729	729	729
Within R^2	0.033	0.024	0.031

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. Promo discount = $1 - p_P/p_F$. Column (3) contains chain-year fixed effects.

related to the eligible share in any specification. These results are consistent with the model's prediction. Proposition 4 characterises the firm's choice of discount depth conditional on operating a promotional channel, but it does not predict that the firm's decision to participate in the channel varies systematically with the eligible share. The empirical evidence is aligned with this distinction. The cannibalisation mechanism operates on the depth of the discount, not on the firm's extensive margin decision to offer promotional tickets. The discount d is the firm's choice variable, whilst the promotion share also reflects the distribution of consumer access costs to the promotional channel, and a movement in the former without a corresponding movement in the latter is consistent with the model.

5.6 Robustness

Table 10 reports estimates across the three measures of the eligible share for all four outcomes. Pass-through to the full price is robust across measures, and the coefficients are significant at 1% for all measures. The promotional discount is also positive and significant across all three measures. Pass-through to PMI is positive across measures and significant for the α^{broad}; the demographic measure estimate is positive but imprecisely estimated, reflecting the smaller cross-municipal variance

Table 9. Proposition 4: eligible share and the promotional ticket share

	Dep. variable: Promo share		
	(1) between	(2) TWFE	(3) chain-year
Eligible share	−0.237	1.500	−0.015
	(0.279)	(0.955)	(0.159)
log income p.c.	−0.016	−0.017	−0.003
	(0.037)	(0.069)	(0.023)
log population	0.007	−0.021	−0.007
	(0.033)	(0.143)	(0.016)
Screens	0.001	0.013	0.004
	(0.007)	(0.014)	(0.005)
N	1036	1036	1036
Within R^2	0.007	0.016	0.004

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. Promo share = share of paid tickets sold at the promotional price. Column (3) contains chain-year fixed effects.

of the demographic measure and the higher noise in PMI as a constructed outcome. The promotion share is null across all three measures, consistent with the model's prediction that the cannibalisation mechanism operates on discount depth rather than on the extensive-margin decision to offer a promotional channel.

Table 10. Robustness across measures of the eligible share

	Dep. var.: log full price			Dep. var.: PMI			Dep. var.: Discount			Dep. var.: Promo share		
Eligible share	0.925***			0.436**			0.297**			−0.015		
	(0.278)			(0.210)			(0.117)			(0.159)		
Eligible share (broad)		0.930***			0.507***			0.181*			0.055	
		(0.279)			(0.196)			(0.109)			(0.171)	
Eligible share (demog.)			1.950***			0.822			0.675**			0.277
			(0.606)			(0.510)			(0.283)			(0.420)
N	1036	1036	1036	1036	1036	1036	729	729	729	1036	1036	1036
Within R^2	0.216	0.222	0.196	0.223	0.232	0.214	0.031	0.017	0.026	0.004	0.005	0.006

Note: * denotes significance at 10%, ** at 5%, and *** at 1%. All columns weighted by paid tickets. Standard errors are clustered by municipality. All columns condition on chain-by-year fixed effects. All specifications contain the control variables log income p.c., log population, and number of screens.

6 Conclusion

In this paper, I study the consequences of a theoretically under-explored form of price regulation: a legally-mandated ratio between the prices charged to eligible and non-eligible consumers. Brazil's half-price law, which applies to cinemas and other cultural activities, offers a clean setting for studying this class of regulation, and the data available for cinema markets makes the empirical quantification possible at a level of detail unavailable in comparable markets.

My model identifies two mechanisms through which the mandated ratio shapes firm pricing. First, a cross-subsidy mechanism: the firm raises the posted full price above its unconstrained level, drawing surplus from non-eligible consumers to subsidise the mandated half-price. Second, a cannibalisation mechanism: because the mandate distorts the full price upward, the firm finds it profitable to redirect non-eligible consumers to a voluntary promotional channel by setting a deeper discount than it would under voluntary price discrimination. The model predicts that, in markets where the mandate binds, both the full price and the voluntary promotional discount should increase with the eligible share.

Using a municipality-year panel of Brazilian cinemas over 2018-2023, I find empirical support for both mechanisms. A one percentage point higher eligible share is associated with a 0.93% higher posted full price, a 0.44% higher average revenue per admission, and a 0.30 percentage point deeper voluntary promotional discount. The pass-through estimates confirm the cross-subsidy mechanism, and the promotional response confirms the cannibalisation mechanism. The empirical analysis also documents that two-thirds of the variation in posted full prices across municipalities is explained by which national chain operates each market rather than by local market conditions.

Two limitations of the present paper point direction for future work. First, the data do not allow me to test the welfare implications of the model. The estimates identify the marginal response of prices to eligible share, but they do not identify the level effect of the mandate. A welfare comparison between the half-price regime and its alternatives is, therefore, left to future work. Second, the monopoly assumption fits the majority of Brazilian municipalities with cinema activity but not the largest markets, where strategic interaction among chains may alter the firm's pricing response. Extending the analysis to oligopolistic markets is a natural next step.

References

Aguirre, I., Cowan, S. and Vickers, J. (2010), 'Monopoly Price Discrimination and Demand Curvature', *American Economic Review* **100**(4), 1601–1615.

Calzada, J. and Fageda, X. (2012), 'Discounts and Public Service Obligations in the Airline Market: Lessons from Spain', *Review of Industrial Organization* **40**(4), 291–312.

Courty, P. and Pagliero, M. (2012), 'The Impact of Price Discrimination on Revenue: Evidence from the Concert Industry', *Review of Economics and Statistics* **94**(1), 359–369.

Cowan, S. (2012), 'Third-Degree Price Discrimination and Consumer Surplus', *The Journal of Industrial Economics* **60**(2), 333–345.

Davis, P. (2006*a*), 'MEASURING THE BUSINESS STEALING, CANNIBALIZATION AND MARKET EXPANSION EFFECTS OF ENTRY IN THE U.S. MOTION PICTURE EXHIBITION MARKET', *The Journal of Industrial Economics* **54**(3), 293–321.

Davis, P. (2006*b*), 'Spatial competition in retail markets: Movie theaters', *The RAND Journal of Economics* **37**(4), 964–982.

de Roos, N. and McKenzie, J. (2014), 'Cheap Tuesdays and the demand for cinema', *International Journal of Industrial Organization* **33**, 93–109.

Einav, L. (2007), 'Seasonality in the U.S. motion picture industry', *The RAND Journal of Economics* **38**(1), 127–145.

Fageda, X., Jiménez, J. L. and Valido, J. (2017), 'An empirical evaluation of the effects of European public policies on island airfares', *Transportation Research Part A: Policy and Practice* **106**, 288–299.

Leslie, P. (2004), 'Price Discrimination in Broadway Theater', *The RAND Journal of Economics* **35**(3), 520.

Mussa, M. and Rosen, S. (1978), 'Monopoly and product quality', *Journal of Economic Theory* **18**(2), 301–317.

Orbach, B. Y. and Einav, L. (2007), 'Uniform prices for differentiated goods: The case of the movie-theater industry', *International Review of Law and Economics* **27**(2), 129–153.

Schmalensee, R. (1981), 'Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination', *American Economic Review* **71**(1), 242–247.

Stole, L. A. (2007), Chapter 34 Price Discrimination and Competition, *in* 'Handbook of Industrial Organization', Vol. 3, Elsevier, pp. 2221–2299.

Varian, H. R. (1985), 'Price Discrimination and Social Welfare', *American Economic Review* **75**(4), 870–875.

Wink Junior, M. V., Ribeiro, F. G., Florissi, S. and Zuanazzi, P. T. (2016), 'Os efeitos da criação de leis de meia entrada para estudantes sobre o consumo de bens e serviços culturais no Brasil', *Estudos Econômicos (São Paulo)* **46**(4), 745–781.

Appendix A. Proofs

Proof of Corollary 1. Define $A(\alpha) = \bar{a}(2-\alpha)$ and $B(\alpha) = \bar{b}(4-3\alpha)$. Then $p^* = A/B$ and:

$$\frac{\partial p^*}{\partial \alpha} = \frac{A'B - AB'}{B^2} = \frac{(-\bar{a})\bar{b}(4-3\alpha) - \bar{a}(2-\alpha)(-3\bar{b})}{\bar{b}^2(4-3\alpha)^2} = \frac{\bar{a}\bar{b}[3(2-\alpha)-(4-3\alpha)]}{\bar{b}^2(4-3\alpha)^2} = \frac{2\bar{a}}{\bar{b}(4-3\alpha)^2}$$

which is strictly positive for all $\alpha \in [0, 1)$. □

Proof of Proposition 6. In the cannibalization section the per-capita-with-share form of the half-price-law full price is

$$p^*_{HL} = \frac{2(1-\alpha_H)\,a_N + \alpha_H\,a_H}{4(1-\alpha_H)\,b_N + \alpha_H\,b_H}.$$

Comparing to $p^*_N = a_N/(2b_N)$ and cross-multiplying by the positive denominators,

$$p^*_{HL} > p^*_N \iff 2b_N\left[2(1-\alpha_H)\,a_N + \alpha_H\,a_H\right] > a_N\left[4(1-\alpha_H)\,b_N + \alpha_H\,b_H\right].$$

The terms $4(1-\alpha_H)a_N b_N$ cancel from both sides, leaving

$$2\alpha_H a_H b_N > \alpha_H a_N b_H.$$

For $\alpha_H > 0$, this is equivalent to $2a_H b_N > a_N b_H$, i.e., $a_H/b_H > a_N/(2b_N)$. □

Proof of Proposition 7. Throughout this proof, $d^*_S(p)$ denotes the separable benchmark discount evaluated at the same fixed full price p (i.e., the discount that would set $p(1-d) = p^*_N$), while d^*_{3T} denotes the separable three-tier model's equilibrium discount.

Part (i): Conditional on the full price. Fix an arbitrary full price $p \in (p^*_N, a_N/b_N)$ (so non-eligible demand is strictly positive, $q_N(p) > 0$) and assume the density of the hassle distribution satisfies $g(h) > 0$ on the relevant interval of switching thresholds. Concretely, $g(p - p^*_N) > 0$, so that small perturbations of d around the separable benchmark induce strictly positive marginal switching. Treat the firm's problem as choosing d to maximise (with total population normalised to $N = 1$)

$$\pi_{NE}(p, d) = (1 - \alpha_H)\Big[(1 - \phi(d))\,\Pi(p) + \phi(d)\,\Pi\big(p(1-d)\big)\Big],$$

where $\Pi(\tilde{p}) = \tilde{p}\, q_N(\tilde{p})$, $q_N(\tilde{p}) = a_N - b_N\,\tilde{p}$, and $\phi(d) = G(d\,p)$ at the fixed p.

Under linear demand Π is strictly concave and maximised at $\tilde{p}^* = a_N/(2b_N) = p_N^*$, with $\Pi(p_N^*) > 0$ since $p_N^* < a_N/b_N$.

Step 1: FOC structure. Differentiating with respect to d:

$$\frac{\partial \pi_{NE}}{\partial d} = (1 - \alpha_H)\Big[\phi(d) \cdot \Pi'\big(p(1-d)\big) \cdot (-p) + \phi'(d) \cdot \big[\Pi\big(p(1-d)\big) - \Pi(p)\big]\Big]. \quad \text{(A.1)}$$

The first term is the intensive margin (effect of a deeper discount on profit from existing promo buyers), and the second is the extensive margin (net profit change from marginal switchers).

Step 2: Evaluate at $d = d_S^*(p)$. The separable-model discount at p is defined by $p(1 - d_S^*(p)) = p_N^*$, which satisfies $\Pi'(p_N^*) = 0$. At $d = d_S^*(p)$:

- The intensive margin vanishes: $\Pi'(p_N^*) = 0 \implies$ first term = 0.
- The extensive margin is strictly positive. Since p_N^* maximises $\Pi(\cdot)$ and $p > p_N^*$ (by hypothesis), we have $\Pi(p_N^*) > \Pi(p)$, and $\phi'(d) = p \cdot g(d\,p) > 0$. Thus:

$$\left.\frac{\partial \pi_{NE}}{\partial d}\right|_{d=d_S^*(p)} = (1 - \alpha_H) \cdot \phi'(d_S^*(p)) \cdot \underbrace{\big[\Pi(p_N^*) - \Pi(p)\big]}_{>0} > 0. \quad \text{(A.2)}$$

Since profit is strictly increasing in d at $d_S^*(p)$, the optimum at this fixed p satisfies $d_C^*(p) > d_S^*(p)$, equivalently $p(1 - d_C^*(p)) < p_N^*$.

Step 3: Interior solution. At $d_C^*(p) > d_S^*(p)$, the promo price $p(1 - d_C^*(p)) < p_N^*$. Here $\Pi'(p(1 - d_C^*(p))) > 0$, so the intensive margin is negative. The optimum balances this loss against the extensive margin, which stays positive as long as $\Pi(p(1-d)) > \Pi(p)$. The interior FOC is:

$$\underbrace{\phi(d_C^*(p)) \cdot \Pi'\big(p(1-d_C^*(p))\big) \cdot (-p)}_{\text{intensive margin} < 0} + \underbrace{\phi'(d_C^*(p)) \cdot \big[\Pi\big(p(1-d_C^*(p))\big) - \Pi(p)\big]}_{\text{extensive margin} > 0} = 0. \quad \text{(A.3)}$$

Existence follows because the derivative is positive at $d = d_S^*(p)$ (Step 2) and as $d \to 1$, $\Pi(p(1-d)) \to \Pi(0) = 0 < \Pi(p)$, so the extensive margin eventually turns negative; by continuity, the FOC holds at some interior $d_C^*(p) \in (d_S^*(p), 1)$.

Part (ii): Equilibrium comparison. Evaluating Part (i) at $p = p_C^*$, the cannibalization equilibrium discount d_C^* is strictly deeper than the separable benchmark discount evaluated at the same full price, $d_S^*(p_C^*) = 1 - p_N^*/p_C^*$.

The comparison to the separable three-tier equilibrium discount $d_{3T}^* = 1 - p_N^*/p_{3T}^*$ (Corollary on the promotional discount rate, identifying $p_P^* = p_N^*$ when $a_P = a_N$ and $b_P = b_N$) is more delicate, because it requires knowing p_C^* versus p_{3T}^*, which depends on which separable benchmark one is comparing against (i.e., what α_P is assumed in the separable model). Therefore, if $p_C^* \geq p_{3T}^*$, then $1 - p_N^*/p_C^* \geq 1 - p_N^*/p_{3T}^* = d_{3T}^*$, and combining with Part (i) at $p = p_C^*$ yields $d_C^* > d_{3T}^*$. Otherwise, the equilibrium-level ranking of discounts depends on the relative magnitudes of the two equilibrium full prices. □

Proof of Proposition 8. By Corollary 3, Case (b) of Proposition 1 implies $p_{HL}^* < p_N^*$. Evaluate the firm's problem at $p = p_{HL}^*$ and consider any $d > 0$ in the interior of the support of G. The FOC for d in (21) contains two terms, each of which is strictly negative under Case (b):

Intensive margin. The first term equals $(1-\alpha_H)\,\phi(d)\,\Pi'\big(p(1-d)\big)\,(-p)$, where $\Pi(\tilde{p}) = \tilde{p}\,q_N(\tilde{p})$. Since $p(1-d) < p < p_N^*$ for $d > 0$, and Π is strictly increasing on $[0, p_N^*]$ (by strict concavity with $\Pi'(p_N^*) = 0$), we have $\Pi'(p(1-d)) > 0$. With $\phi(d) > 0$ and $-p < 0$, the intensive margin is strictly negative.

Extensive margin. The second term equals $(1-\alpha_H)\,\phi'(d)\,[\Pi(p(1-d)) - \Pi(p)]$. Since Π is strictly increasing on $[0, p_N^*]$ and $p(1-d) < p < p_N^*$, we have $\Pi(p(1-d)) < \Pi(p)$. With $\phi'(d) = p\,g(d\,p) > 0$, the extensive margin is strictly negative.

Both terms are strictly negative for any $d > 0$ where $\phi(d) > 0$ and $\phi'(d) > 0$, so $\partial\pi/\partial d < 0$ throughout the interior of the support. The unique optimum is therefore the corner $d_C^* = 0$. With no switching at $d = 0$, the full price reduces to the baseline two-tier optimum p_{HL}^*. □

Proof of Proposition 9. Normalising total population to $N = 1$, treat ϕ as an exogenous parameter and let d adjust optimally for each (p, ϕ), so the firm's problem reduces to choosing p to maximise

$$\tilde{\pi}(p;\phi) = (1-\alpha_H)\Big[(1-\phi)\,\Pi(p) + \phi\,\Pi\big(\tilde{p}^*(p)\big)\Big] + \alpha_H\,\frac{p}{2}\,q_H(p/2),$$

where $\Pi(\tilde{p}) = \tilde{p}\,q_N(\tilde{p})$ and $\tilde{p}^*(p) = \arg\max_{\tilde{p}\le p}\Pi(\tilde{p})$. The interior region of Proposition 6 guarantees $\tilde{p}^*(p) < p$ at $p = p_{HL}^*$, so $\tilde{p}^*(p) = a_N/(2b_N) = p_N^*$ is invariant to p locally.

The FOC for p at fixed ϕ is:

$$\Psi(p,\phi) \equiv (1-\alpha_H)(1-\phi)\,\Pi'(p) + \tfrac{1}{2}\alpha_H\,(a_H - b_H\,p) = 0,$$

where the promo term vanishes because $\partial\Pi(\tilde{p}^*)/\partial p = 0$ locally. The second-order condition $\Psi_p < 0$ holds at any interior optimum. By the implicit function theorem,

$$\frac{d\,p^*}{d\phi} = -\frac{\Psi_\phi}{\Psi_p} = \frac{(1-\alpha_H)\,\Pi'(p^*)}{\Psi_p}.$$

In the interior region $p_C^* > p_N^*$, so $\Pi'(p^*) < 0$ and $\Psi_p < 0$; hence $d\,p^*/d\phi > 0$. At $\phi = 0$ the FOC is the baseline two-tier FOC (under the per-capita identification $a_F = a_N$, $b_F = b_N$), delivering $p_C^*(0) = p_{HL}^*$. At $\phi = 1$ the FOC reduces to $\tfrac{1}{2}\alpha_H\,(a_H - b_H\,p) = 0$, yielding $p = a_H/b_H$. Monotonicity in ϕ along the interior branch delivers $p_C^*(\phi) > p_{HL}^*$ for $\phi > 0$. □

Proof of Proposition 10. Compute $L_V - L_{HL}$:

$$L_V - L_{HL} = \frac{a_F^2 b_H + a_H^2 b_F}{8 b_F b_H} - \frac{(2a_F + a_H)^2}{8(4b_F + b_H)} \tag{A.4}$$

Placing over a common denominator $8 b_F b_H (4b_F + b_H)$:

$$\text{Numerator} = (a_F^2 b_H + a_H^2 b_F)(4b_F + b_H) - b_F b_H (2a_F + a_H)^2 \tag{A.5}$$

Expanding the first product:

$$4a_F^2 b_F b_H + a_F^2 b_H^2 + 4a_H^2 b_F^2 + a_H^2 b_F b_H$$

Expanding the second:

$$b_F b_H (4a_F^2 + 4a_F a_H + a_H^2) = 4a_F^2 b_F b_H + 4a_F a_H b_F b_H + a_H^2 b_F b_H$$

Subtracting:

$$a_F^2 b_H^2 + 4a_H^2 b_F^2 - 4a_F a_H b_F b_H = (a_F b_H - 2a_H b_F)^2$$

Therefore:

$$W_{HL} - W_V = \frac{(a_F b_H - 2a_H b_F)^2}{8\, b_F b_H (4b_F + b_H)} \geq 0 \tag{A.6}$$

with equality if and only if $a_F b_H = 2a_H b_F$. □

Proof of Proposition 11. The welfare difference is $\Delta W = L_U - L_{HL}$. Compute:

$$L_U - L_{HL} = \frac{(a_F + a_H)^2}{8(b_F + b_H)} - \frac{(2a_F + a_H)^2}{8(4b_F + b_H)} \tag{A.7}$$

Over a common denominator $8(b_F + b_H)(4b_F + b_H)$:

$$\text{Num.} = (a_F + a_H)^2 (4b_F + b_H) - (2a_F + a_H)^2 (b_F + b_H) \tag{A.8}$$

Expanding term by term:

$$(a_F+a_H)^2(4b_F+b_H) = 4a_F^2 b_F + a_F^2 b_H + 8a_F a_H b_F + 2a_F a_H b_H + 4a_H^2 b_F + a_H^2 b_H$$
$$(2a_F+a_H)^2(b_F+b_H) = 4a_F^2 b_F + 4a_F^2 b_H + 4a_F a_H b_F + 4a_F a_H b_H + a_H^2 b_F + a_H^2 b_H$$

Subtracting:

$$-3a_F^2 b_H + 4a_F a_H b_F - 2a_F a_H b_H + 3a_H^2 b_F = b_F a_H(4a_F + 3a_H) - b_H a_F(3a_F + 2a_H)$$

The condition $\Delta W > 0$ requires this expression to be positive, yielding (27). Dividing by $b_F a_F^2$ gives the threshold in terms of ρ and σ. □

Proof of Corollary 6. From the derivation of $Q_{HL} - Q_U$, $\Delta Q = (2a_H b_F - a_F b_H)/\big(2(4b_F + b_H)\big)$. The sign depends on $2a_H b_F \gtrless a_F b_H$, i.e., $\sigma \lessgtr 2\rho$. To verify $\bar{\sigma}(\rho) < 2\rho$: $\rho(4 + 3\rho)/(3 + 2\rho) < 2\rho$ if and only if $(4 + 3\rho) < 2(3 + 2\rho) = 6 + 4\rho$ if and only if $\rho > -2$, which always holds. □

Proof of Proposition 14. The firm chooses (p, d) jointly. The two first-order conditions are

$$\Psi(p, d, \alpha_H) \equiv \frac{\partial \pi}{\partial p} = \frac{\alpha_H}{2} M_H\left(\frac{p}{2}\right) + (1 - \alpha_H)\,\widetilde{\Psi}_{F+P}(p, d) = 0, \tag{A.9}$$

$$H(p, d, \alpha_H) \equiv \frac{\partial \pi}{\partial d} = (1 - \alpha_H)\, h(p, d) = 0, \tag{A.10}$$

where $h(p, d)$ collects the non-eligible-side derivative (and is independent of α_H as a consequence of the additive separability of the eligible term with respect to d). The second-order condition is that the Hessian

$$\mathbf{H} = \begin{pmatrix} \pi_{pp} & \pi_{pd} \\ \pi_{dp} & \pi_{dd} \end{pmatrix}$$

is negative definite at the optimum, so $\det \mathbf{H} > 0$ and $\pi_{pp}, \pi_{dd} < 0$.

Totally differentiating the system $\Psi = 0$, $H = 0$ in α_H:

$$\mathbf{H}\begin{pmatrix} dp^*/d\alpha_H \\ dd^*/d\alpha_H \end{pmatrix} = -\begin{pmatrix} \Psi_{\alpha_H} \\ H_{\alpha_H} \end{pmatrix},$$

where the right-hand entries denote partial derivatives in α_H at fixed (p, d). From (A.10), $H = (1 - \alpha_H)h$ and $H_{\alpha_H} = -h(p, d)$. At the interior optimum $h(p^*, d^*) = 0$, so

$$H_{\alpha_H}(p^*, d^*, \alpha_H) = 0. \tag{A.11}$$

With $H_{\alpha_H} = 0$, the system collapses to

$$\begin{pmatrix} dp^*/d\alpha_H \\ dd^*/d\alpha_H \end{pmatrix} = -\mathbf{H}^{-1}\begin{pmatrix} \Psi_{\alpha_H} \\ 0 \end{pmatrix} = \frac{1}{\det \mathbf{H}}\begin{pmatrix} -\pi_{dd}\Psi_{\alpha_H} \\ \pi_{dp}\Psi_{\alpha_H} \end{pmatrix}.$$

Since $\det \mathbf{H} > 0$ and $-\pi_{dd} > 0$,

$$\operatorname{sign}\left(\frac{dp^*}{d\alpha_H}\right) = \operatorname{sign}(\Psi_{\alpha_H}) = \operatorname{sign}\left(\frac{1}{2}M_H\left(\frac{p^*}{2}\right) - \widetilde{\Psi}_{F+P}(p^*, d^*)\right), \tag{A.12}$$

using $\Psi_{\alpha_H} = \frac{1}{2}M_H(p^*/2) - \widetilde{\Psi}_{F+P}(p^*, d^*)$ from differentiating (A.9).

The FOC (A.9) requires $M_H(p^*/2)$ and $\widetilde{\Psi}_{F+P}(p^*, d^*)$ to have opposite signs (since $\alpha_H, 1 - \alpha_H > 0$). When $p^*/2 < p^*_H$, $M_H(p^*/2) > 0$, which forces $\widetilde{\Psi}_{F+P}(p^*, d^*) < 0$. Then

$$\frac{1}{2}M_H\left(\frac{p^*}{2}\right) - \widetilde{\Psi}_{F+P}(p^*, d^*) > 0,$$

so $dp^*/d\alpha_H > 0$. □

Proof of Proposition 15. Let $H(p, d, \alpha_H) \equiv \partial\pi/\partial d = 0$ denote the first-order condition for d. Since the half-price group's demand q_H does not depend on d, the function H satisfies:

$$H(p, d, \alpha_H) = (1 - \alpha_H)\,h(p, d), \tag{A.13}$$

where $h(p,d) \equiv \frac{\partial}{\partial d}\left[p\,(1-\phi)\,s_N(p) + p(1-d)\,\phi\,s_N(p(1-d))\right]$ is independent of α_H. It follows that at any interior optimum (p^*, d^*) where $h(p^*, d^*) = 0$:

$$H_\alpha(p^*, d^*, \alpha_H) \equiv \frac{\partial H}{\partial \alpha_H} = -h(p^*, d^*) = 0. \tag{A.14}$$

Totally differentiating $H = 0$ along the equilibrium path with respect to α_H:

$$H_p \frac{dp^*}{d\alpha_H} + H_d \frac{dd^*}{d\alpha_H} + H_\alpha = 0 \implies \frac{dd^*}{d\alpha_H} = -\frac{H_p}{H_d}\frac{dp^*}{d\alpha_H}, \tag{A.15}$$

using $H_\alpha = 0$ at the equilibrium. The second-order condition requires $H_d \equiv \partial^2\pi/\partial d^2 < 0$. By hypothesis $dp^*/d\alpha_H > 0$, so:

$$\text{sign}\left(\frac{dd^*}{d\alpha_H}\right) = \text{sign}(H_p) = \text{sign}\left(\frac{\partial^2 \pi}{\partial d\,\partial p}\right). \tag{A.16}$$

The result therefore reduces to signing the cross-partial H_p. The proposition takes $H_p > 0$ as a maintained condition rather than deriving it from primitives. The economic rationale, discussed in the main text, is that when the full price p rises (because α_H has risen) and exceeds the non-eligible standalone optimum, the per-consumer profit gap between the promo and full-price tiers widens, increasing the marginal incentive to deepen the discount. Under the maintained $H_p > 0$ and the standard second-order condition $H_d < 0$, together with $dp^*/d\alpha_H > 0$, we have $dd^*/d\alpha_H > 0$. □

Proof of Proposition 16. The argument follows the proof of Proposition 15 up to the sign of $dp^*/d\alpha_H$. From the implicit-function calculation, at any interior optimum:

$$\frac{dd^*}{d\alpha_H} = -\frac{H_p}{H_d}\frac{dp^*}{d\alpha_H}. \tag{A.17}$$

The second-order condition gives $H_d < 0$, and the maintained cross-partial assumption gives $H_p > 0$, so $-H_p/H_d > 0$. By hypothesis of the proposition, $dp^*/d\alpha_H < 0$ (the logit Case (b) regime characterised in Remark 6). Therefore $dd^*/d\alpha_H < 0$. □

Proof of Proposition 17. By the chain rule:

$$\frac{d\phi(d^*, p^*)}{d\alpha_H} = \underbrace{\frac{\partial\phi}{\partial d}}_{=k\phi(1-\phi)p>0} \cdot \underbrace{\frac{dd^*}{d\alpha_H}}_{>0} + \underbrace{\frac{\partial\phi}{\partial p}}_{=k\phi(1-\phi)d>0} \cdot \underbrace{\frac{dp^*}{d\alpha_H}}_{>0} > 0, \tag{A.18}$$

where $dd^*/d\alpha_H > 0$ follows from Proposition 15 and $dp^*/d\alpha_H > 0$ from Proposition 14. □

Proof of Proposition 18. By the chain rule:

$$\frac{d\phi(d^*, p^*)}{d\alpha_H} = \underbrace{\frac{\partial\phi}{\partial d}}_{=k\phi(1-\phi)p>0} \cdot \underbrace{\frac{dd^*}{d\alpha_H}}_{<0} + \underbrace{\frac{\partial\phi}{\partial p}}_{=k\phi(1-\phi)d>0} \cdot \underbrace{\frac{dp^*}{d\alpha_H}}_{<0} < 0, \tag{A.19}$$

where $dd^*/d\alpha_H < 0$ follows from Proposition 16 and $dp^*/d\alpha_H < 0$ from Remark 6. □